

Process control laboratory practice

Interacting control loops

INTRODUCTION

There can be several controlled variables and several manipulated variables in a process. In this case we speak about Multiple Input – Multiple Output (MIMO) systems.

If the process can be decomposed to pairwise independent processes then the MIMO problem is separated into several SISO problems. However, in the general case the process cannot be perfectly decomposed, and some or all of the controlled variables depend on some or all the possible manipulated variables.

Suppose now that we have N controlled variables c_1, c_2, \dots, c_N , and N manipulated variables m_1, m_2, \dots, m_N such that this set has some influence to all the controlled variables. Then the process can be represented by some functional

$$\{c_1, c_2, \dots, c_N\} = \mathbf{Process}[\{m_1, m_2, \dots, m_N\}]$$

Then one has to construct a control system that can be represented in the same way:

$$\{m_1, m_2, \dots, m_N\} = \mathbf{Control}[\{c_1, c_2, \dots, c_N\}]$$

This is a rather difficult task not just because of its tuning but because of the many possible structures that can be considered. A simpler approach is selecting N (c_i, m_j) pairs so that each controlled variable takes place in exactly one such a pair and each manipulated variable takes place in exactly one such a pair, and construct a control loop to each pair. The number of possible matchings is $N! = 1 \cdot 2 \cdot \dots \cdot (N-1) \cdot N$ which is a large number if N is large but yet much smaller than the number of possibilities otherwise, and the general properties of such single loops already are known. This is **distributed control**.

Relative gains

Consider a system of (m_1, m_2) and (c_1, c_2) . The transfer function of the process can be represented by $2 \times 2 = 4$ simpler transfer functions $G_{ij}(s)$:

$$C_1(s) = G_{11}(s) \cdot M_1(s) + G_{12}(s) \cdot M_2(s)$$

$$C_2(s) = G_{21}(s) \cdot M_1(s) + G_{22}(s) \cdot M_2(s)$$

There are only $2! = 1 \cdot 2 = 2$ possible distributed control systems in this case:

Sys-A:

Loop 1: $m_1 \rightarrow c_1$

Loop 2: $m_2 \rightarrow c_2$

Sys-B:

Loop 1: $m_1 \rightarrow c_2$

Loop 2: $m_2 \rightarrow c_1$

Before deciding which is better, we have to measure or calculate how the loops interact.

Loops' interaction is can be measured by relative gains as the simplest tool.

Relative gain is a ratio of gains A_{ij} of **the same pair** (c_i, m_j) in two different cases:

1. $(A_{ij})_m$ ('manual'): No control is applied, i.e. not a single control loop is switched on.
2. $(A_{ij})_c$ ('controlled'): All the other manipulated variables are somehow kept constant (e.g. by switching on all the other loops and applying good tuning), but just the actual (c_i, m_j) loop is not.

Thus,

$$(A_{ij})_m = \left(\frac{\partial c_i}{\partial m_j} \right)_{m_{k \neq j}}$$

i.e. partial derivative of c_j respect to m_j while all other **manipulated** variables remain constant.

and

$$(A_{ij})_c = \left(\frac{\partial c_i}{\partial m_j} \right)_{c_{h \neq i}}$$

i.e. partial derivative of c_j respect to m_j while all other **controlled** variables remain constant.

Definition of the relative gain λ_{ij} is

$$\lambda_{ij} = \frac{(A_{ij})_m}{(A_{ij})_c} = \frac{\left(\frac{\partial c_i}{\partial m_j} \right)_{m_{k \neq j}}}{\left(\frac{\partial c_i}{\partial m_j} \right)_{c_{h \neq i}}}$$

Meaning of the relative gain

- $\lambda=1$: switching on the other control loops does not influence the gain. If in a MIMO configuration all the relative gains are $\lambda=1$ then there is no interaction between them. This would be the ideal case.
- $\lambda=0$: the gain itself $(A_{ij})_m=0$, i.e. m_j does not influence c_i at all. Such a case is a wrong selection, cannot be used for control.
- $\lambda<0$: the gain changes sign if the other loops are switched on. This is a dangerous selection, cannot be used for control.
- $0<\lambda<1$: switching on the other control loops decreases the gain. This is called positive interaction.
- $\lambda>1$: switching on the other control loops increases the gain. This is called negative interaction. This increases the chance of instability.

Relative gain array

The gains can be arranged in a matrix. For example, in the 2×2 system, we have a 2×2 matrix:

λ_{11}	λ_{12}
λ_{21}	λ_{22}

It can be shown that rows and columns sum up to unity: $\sum_j \lambda_{ij} = 1$ and $\sum_i \lambda_{ij} = 1$.

Binary (2×2) systems

In this case all the relative gains can be calculated based merely on the control-less (m-) gains. Given $A_{11}=(A_{11})_{m2}$, $A_{12}=(A_{12})_{m1}$, $A_{21}=(A_{21})_{m2}$, $A_{22}=(A_{22})_{m1}$, we can write

$$\lambda_{11} = \frac{A_{11} \cdot A_{22}}{A_{11} \cdot A_{22} - A_{12} \cdot A_{21}} = \frac{1}{1 - \frac{A_{12} \cdot A_{21}}{A_{11} \cdot A_{22}}}$$

(as can be derived from the system

$$\begin{aligned} \hat{c}_1 &= A_{11} \cdot \hat{m}_1 + A_{12} \cdot \hat{m}_2 \\ \hat{c}_2 &= A_{21} \cdot \hat{m}_1 + A_{22} \cdot \hat{m}_2 \end{aligned} .)$$

Then by the summing relations, we have

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11} \quad \text{and} \quad \lambda_{22} = \lambda_{11}$$

Decoupling

For successful control, highly interacting loops have to be decoupled.

Selecting matches with RGA being near unit matrix is the best way, and called implicit decoupling.

Another possibility is detuning, i.e. setting less strict tuning of one or more loops.

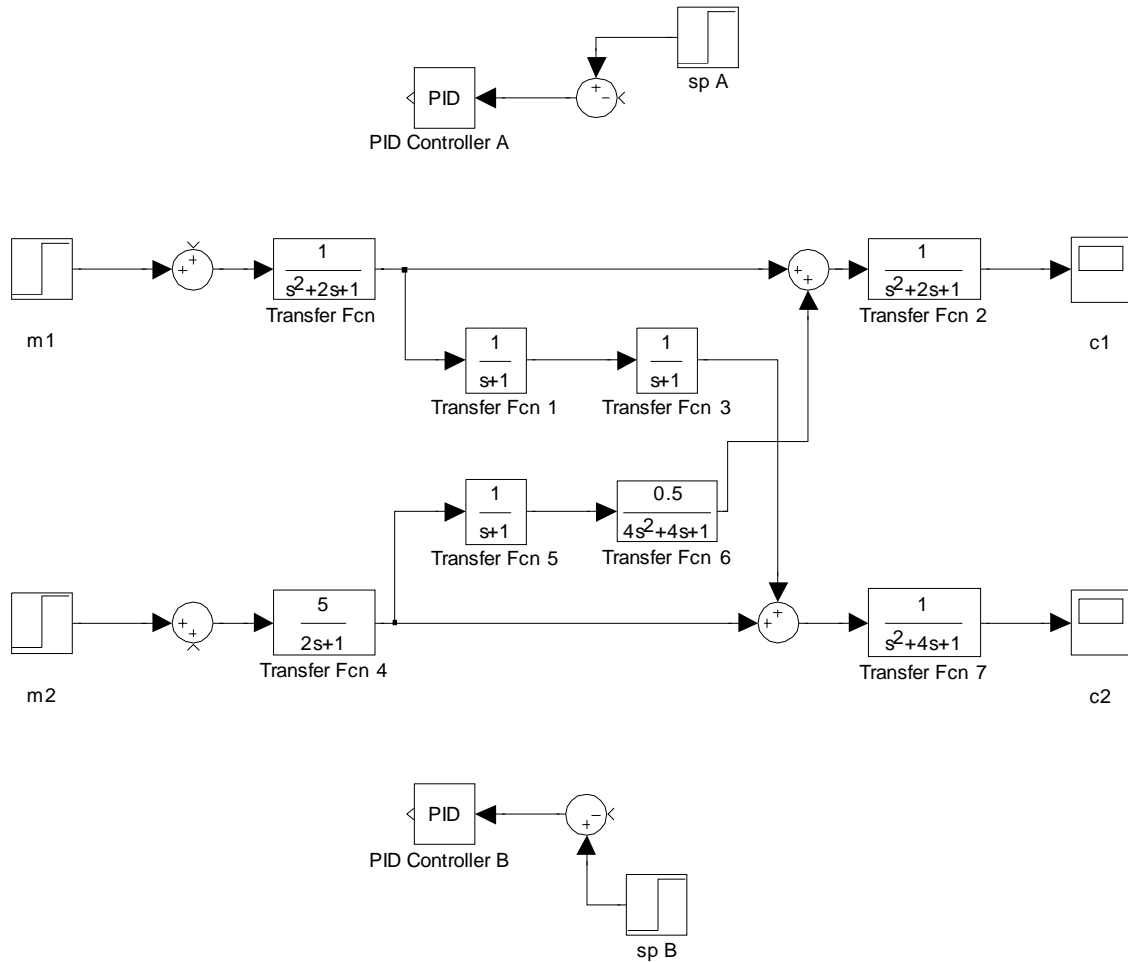
In some fortunate cases the interaction can be compensated by inserting explicit decoupling elements into the control structure. Due to the complex nature of the processes, it is usually done in computer control.

Aim of the practice

1. Measure relative gains.
2. Couple signals for distributed control.
3. Tune interacting feed-back control loops.

Use PI controllers only!

Applied model



Model applied in Simulink

c1, c2: control signals
 m1, m2: modified variables
 spA, spB: set points

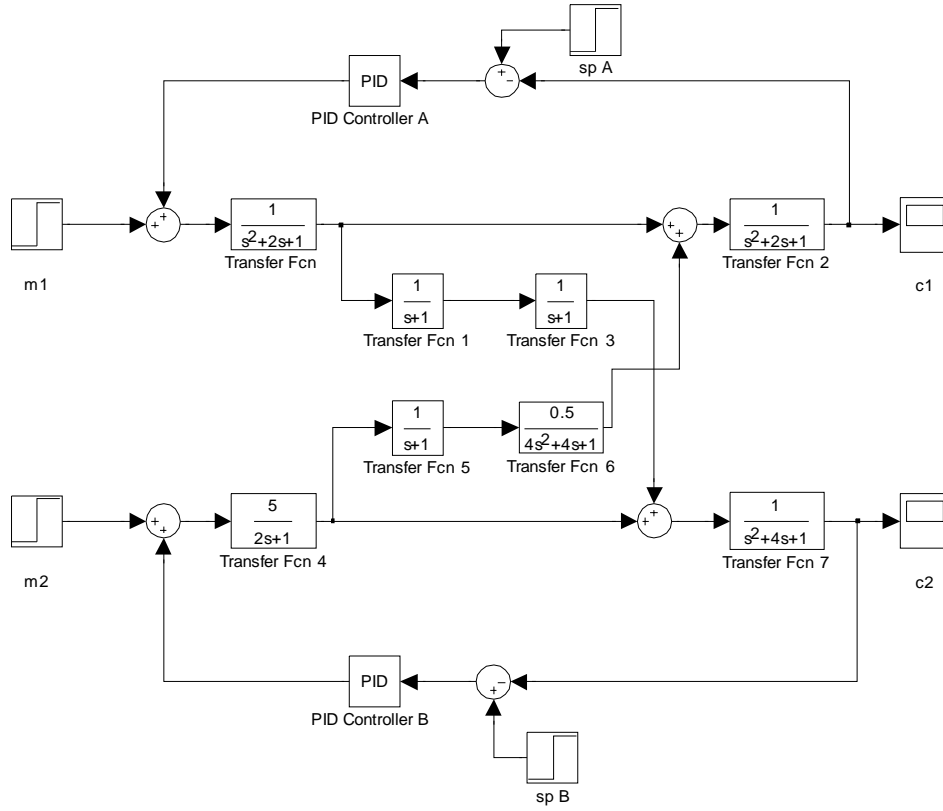
Each controlled signal depends on both modified variables.

$$C1 = (m1 * F + m2 * F4 * F5 * F6) * F2$$

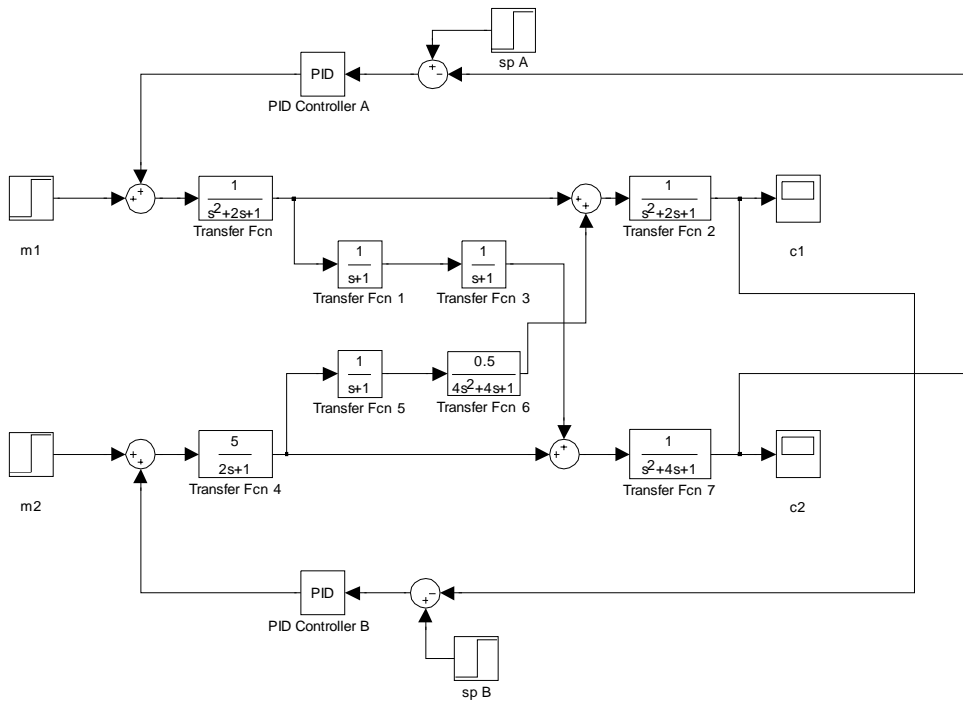
$$C2 = (m2 * F4 + m1 * F * F1 * F3) * F7$$

There are two controlles: A and B.

The two possible distributed control systems are shown in the next page.



Sys A: 1 to 1, 2 to 2



Sys B: 2 to 1, 1 to 2

1. Measure relative gains

1.1. Measure static gains without control:

Table 1. Static gains without control

given values		measured		calculated	
m1	m2	c1	c2		
	0			$A_{11} =$	$A_{21} =$
0				$A_{12} =$	$A_{22} =$

1.2. Calculate relative gains this way:

$$\lambda_{11} = \frac{A_{11} \cdot A_{22}}{A_{11} \cdot A_{22} - A_{12} \cdot A_{21}} = \frac{1}{1 - \frac{A_{12} \cdot A_{21}}{A_{11} \cdot A_{22}}}$$

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11} \quad \text{and} \quad \lambda_{22} = \lambda_{11}$$

Relative gains according to **Table 1**

λ_{ij}	m1	m2
c1		
c2		

1.3. Measure each relative gain directly! (Numerators are already measured, see Table 1.) For measuring denominators, control must be used. Tune the controller with oscillating method. Denote the measured gains by B.

Relative gains when the other controlled variable is kept constant

B_{ij}	m1	m2
c1		
c2		

Relative gains measured directly

λ_{ij}	m1	m2
c1		
c2		

2. Assign coupling.

Select a proper coupling! (Write X to the selected couples!)

Coupling

	m1	m2
c1		
c2		

3. Tune the loops

- Switch on both loops, and type in the tuning parameters according to those determined in step 1.3! Observe how the loops behave!
- If the loops are not stable then re-tune one of the loops while keepin the parameters of the other one.