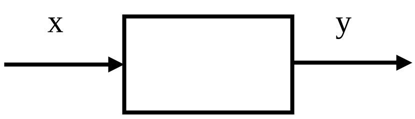
Chemical Process Control
Auxiliary material for hand calculation practices

GENERAL NOTIONS

Notation

t = time
T = time constant
s = complex variable

Processes



The process is 'known' if function $y(t) = f[x(t)]$ is known.
Equivalent information is given by its transfer function G(s) in Laplace domain:
$$Y(s) = G(s) \cdot X(s)$$

Laplace transformation

It transforms any linear differential equation to an ordinary equation.

Two-sided Laplace transformation: $F(s) = \mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) \cdot e^{-s \cdot t} \cdot dt$

One-sided Laplace transformation: $F(s) = \mathcal{L}[f(t)] = \int_{0}^{\infty} f(t) \cdot e^{-s \cdot t} \cdot dt$, this one is simpler if f(x)=0 at t<0. This condition is achieved if there is (was) steady state before t=0 and deviation variable are used: $\hat{x}(t) = x(t) - \overline{x}(t)$

$$f(t) \xrightarrow{\mathcal{L}} \underbrace{F(s) \xrightarrow{\text{calculation}} G(s)}_{\text{used in our calc. practices}} \xrightarrow{\mathcal{L}^{-1}} g(t)$$

Linearity:

$$a \cdot f(t) + b \cdot g(t) \xrightarrow{\mathcal{L}} a \cdot F(s) + b \cdot G(s)$$

Useful particular cases:

$$y(t) \xrightarrow{\mathcal{L}} Y(s)$$

$$\frac{dy(t)}{dt} \xrightarrow{\mathcal{L}} s \cdot Y(s)$$

$$\int y(t) dt \xrightarrow{\mathcal{L}} \frac{X(s)}{s}$$

$$\left(\delta(t) = \frac{d1(t)}{dt} \right) \xrightarrow{\mathcal{L}} 1$$ (unit impulse)

$$1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$$ (unit step)

$$\left(t = \int 1(t) dt \right) \xrightarrow{\mathcal{L}} \frac{1}{s^2}$$ (unit slope ramp)

$$e^{a \cdot t} \xrightarrow{\mathcal{L}} \frac{1}{s - a}$$

I.-II. ORDER ELEMENTS

Proportional element

$$y(t) = A \cdot x(t)$$

$$G(s) = \frac{Y(s)}{X(s)} = A$$

First order element

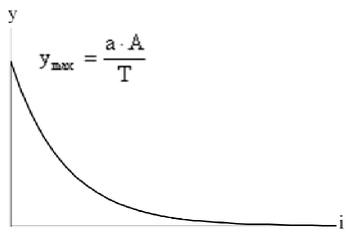
Differential equation: $T\frac{dy(t)}{di} + y(t) = A \cdot x(t)$

Transfer function: $G(s) = \frac{A}{Ts+1}$

Impulse response: $\hat{y} = \frac{a \cdot A}{T} \cdot e^{-\frac{t}{T}}$

Step response: $\hat{y} = a \cdot A \cdot \left[1 - e^{-\frac{t}{T}}\right]$

Ramp response: $\hat{y} = a \cdot A \cdot T \cdot \left[\frac{t}{T} + e^{-\frac{t}{T}} - 1\right]$

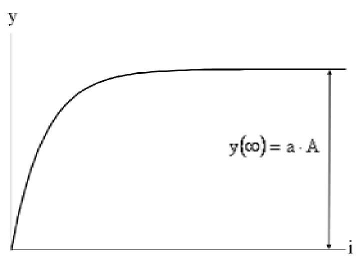


here i stands for t

Its step response:

$$Y(s) = X(s) \cdot G(s) = \frac{a}{s} \cdot \frac{A}{T \cdot s + 1}$$

$$\hat{y}(i) = a \cdot A \cdot \left[1 - e^{-\frac{t}{T}}\right]$$



here i stands for t

Identification from step response

Gain

From how high the response signal increases (in limit)

Time constant: several ways you can do it

1. Sunstitution

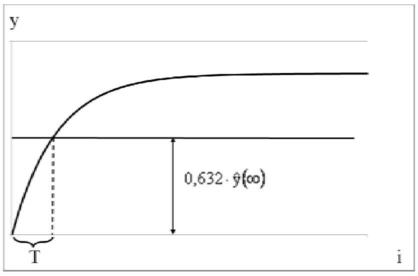
   From any t–y(t) value pair (a point in the plane) you can calculate it. This is subject to errors.

2. Ratio calculation

   If t=T then

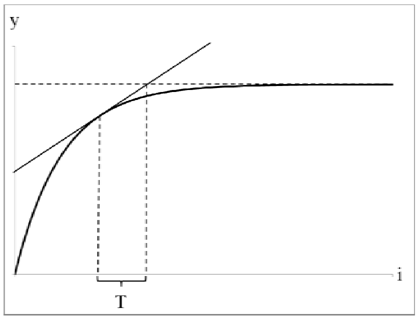
   $$\hat{y}(T) = a \cdot A \cdot \left[1 - e^{-\frac{T}{T}}\right] = \hat{y}(\infty) \cdot \left[1 - e^{-1}\right]$$

   $$\frac{\hat{y}(T)}{\hat{y}(\infty)} = 1 - e^{-1} \approx 0.632$$



3. Slope fitting

   (To any point)



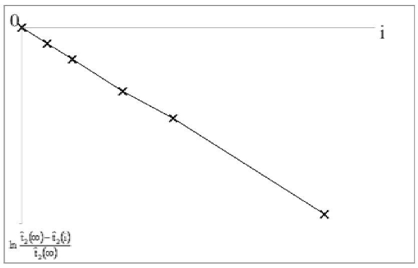
4. Linearization

   $$\hat{y}(t) = a \cdot A \cdot \left[1 - e^{-\frac{t}{T}}\right] = \hat{y}(\infty) \cdot \left[1 - e^{-\frac{t}{T}}\right]$$

   $$\frac{\hat{y}(i)}{\hat{y}(\infty)} = 1 - e^{-\frac{t}{T}}$$

   $$\frac{\hat{y}(\infty) - \hat{y}(t)}{\hat{y}(\infty)} = e^{-\frac{t}{T}}$$

   $$\ln\left[\frac{\hat{y}(\infty) - \hat{y}(t)}{\hat{y}(\infty)}\right] = -\frac{1}{T} \cdot t$$



   The process is of first order if the points fall to a straight line, otherwise its behaviour is different.

   Slope of the straight line: $-\frac{1}{T}$.

   This method is best for cancelling measurement errors.

Examples

jacketed vessel, mixed vessel, CSTR, thermometer

Second order element

Differential equations:

$$T_2^2 \frac{dy^2(t)}{dt^2} + T_1 \frac{dy(t)}{dt} + y(t) = A \cdot x(t) \qquad T^2 \frac{dy^2(t)}{dt^2} + 2\xi T \frac{dy(t)}{dt} + y(t) = A \cdot x(t)$$

Transfer functions:

$$G(s) = \frac{A}{T_2^2 s^2 + T_1 s + 1} \qquad G(s) = \frac{A}{T^2 s^2 + 2\xi T s + 1}$$

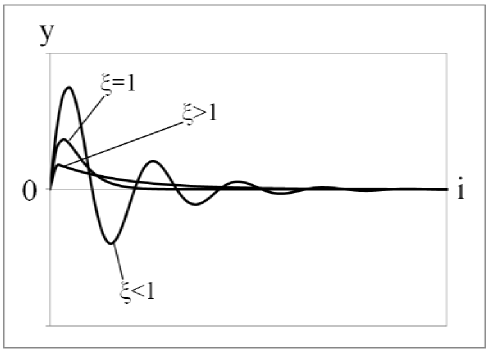
	Impulse responses	Step responses
ξ<1	$\hat{y} = a \cdot A \cdot \left[\frac{1}{\omega T^2} e^{\alpha t} \sin(\omega t)\right]$ where $\alpha = \frac{\xi}{T}$ and $\omega = \frac{1}{T}\sqrt{1-\xi^2}$	$\hat{y} = a \cdot A \cdot \left[1 - e^{\alpha t}\left(\cos(\omega t) + \frac{\alpha}{\omega}\sin(\omega t)\right)\right]$ where $\alpha = \frac{\xi}{T}$ and $\omega = \frac{1}{T}\sqrt{1-\xi^2}$
ξ=1	$\hat{y} = a \cdot A \cdot \left[\frac{1}{T^2} \cdot t \cdot e^{-\frac{t}{T}}\right]$	$\hat{y} = a \cdot A \cdot \left[1 - e^{-\frac{t}{T}}\left(1 + \frac{1}{T}\right)\right]$
ξ>1	$\hat{y} = a \cdot A \cdot \frac{1}{T_1 - T_2}\left[e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}}\right]$	$\hat{y} = a \cdot A \cdot \left[1 - \frac{1}{T_1 - T_2}\left(T_1 \cdot e^{-\frac{t}{T_1}} - T_2 \cdot e^{-\frac{t}{T_2}}\right)\right]$

ξ<1 underdamped, ξ=1 critically damped, ξ>1 overdamped

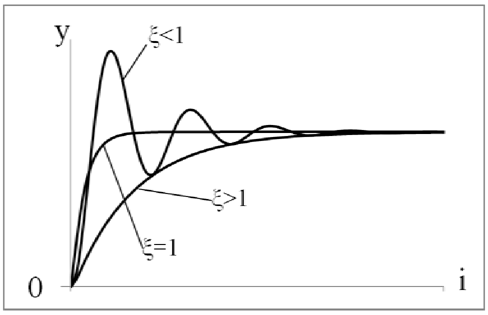
Two basic classes:

1. intrinsic second order elements
2. elements obtained by consecutively connecting two first order elements
   - with equal time constants: ξ=1
   - with unequal time constants: ξ>1

Its impulse response:



Its step response:



VALVES

Maximum throughput at given pressure drop on the valve

definition of $k_{v,max}$: $W_{max} = k_{v,max}\sqrt{\frac{\Delta p_{rel}}{\rho_{rel}}}$

where $k_{v,max}$ – water flow rate throught perfectly open valve for 1 bar pressure drop over it.

definitions: $\Delta p_{rel} = \frac{\Delta p_{valve}}{\Delta p_{atm}} = \frac{\Delta p_{valve}}{1bar}$ $\rho_{rel} = \frac{\rho}{\rho_{water,20°C}} = \frac{\rho}{1000\,\frac{kg}{m^3}}$

W_{max} is not constant but depends on pressure drop and density!

Throuput characteristics:

Linear: $\frac{W}{W_{max}} = \frac{H}{H_{max}} = h$ (this is definition of h)

Square root: $\frac{W}{W_{max}} = \sqrt{\frac{H}{H_{max}}} = \sqrt{h}$

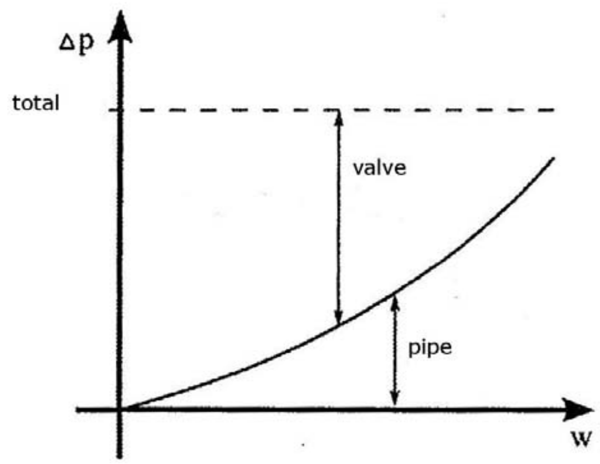
Exponential: $\frac{W}{W_{max}} = \frac{1}{e^n} \cdot e^{n \cdot \frac{H}{H_{max}}} = \frac{1}{e^n} \cdot e^{n \cdot h}$ (n=1..3)

Non-linearity of the process can be offset with properly selected valves.

Valve in pipeline:

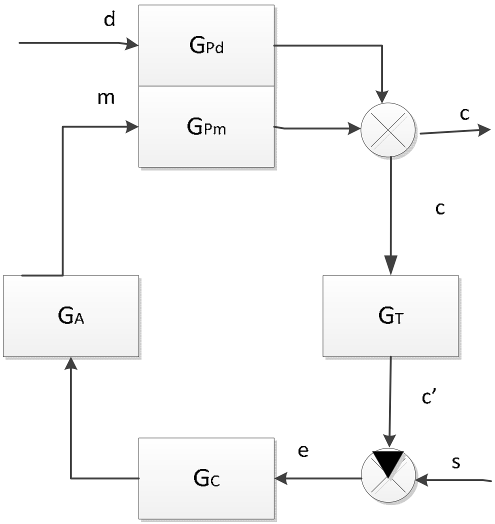
Total pressure drop: $\Delta p_{total} = \Delta p_{pipe} + \Delta p_{valve}$

Due to friction: $\Delta p_{valve} = B \cdot W^2$ (assume turbulent flow)

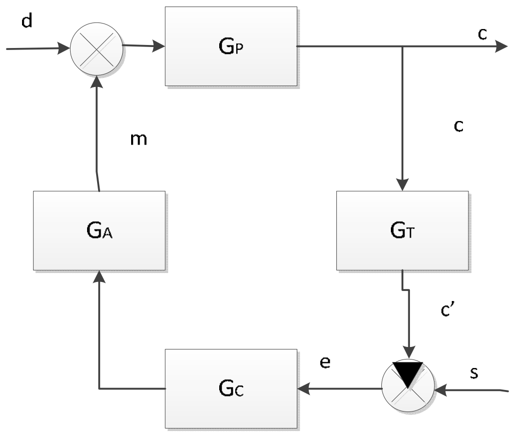


CONTROL LOOPS

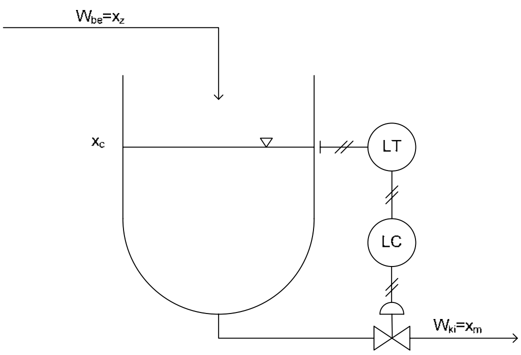
Block scheme

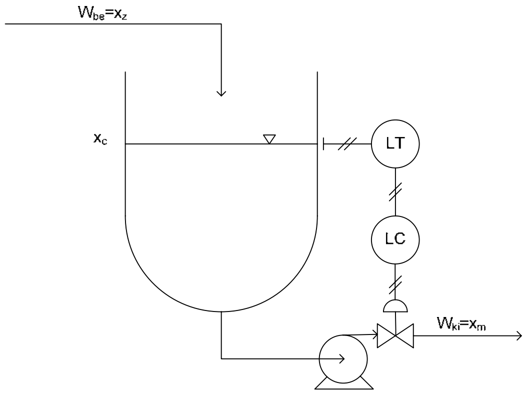


If the process reacts to d as m, then a simplified block scheme can be used:



Level control





Outflow flow rate is manipulated variable, and inflow flow rate is disturbance.

Process:

Free outflow tank

first order element

$$G_P(s) = \frac{h(s)}{W_{in}(s)} = \frac{A}{T \cdot s + 1}$$

Forced outflow tank

integrating element

$$G_P(s) = \frac{h(s)}{W_{in}(s)} = \frac{A}{s}$$

h denotes level (height), B denotes cross section area

$$A = \frac{2 \cdot h}{W_{in}} \qquad T = \frac{2 \cdot B \cdot h}{W_{in}}$$

$$A = \frac{1}{B}$$

Transmitter – Proportional element

Controller – P controller (proportional element)

Actuator – Valve with linear working characteristic, proportional element

Closed loop transfer functions:

Forced outflow tank

Input flow rate $\Rightarrow$ liquid level:

$$G^*(s)=\frac{h(s)}{W_{in}(s)}=\frac{c(s)}{d(s)}=\frac{G_P(s)}{1+G_P(s)\cdot G_{Tr}(s)\cdot G_C(s)\cdot G_{Ac}(s)}=\frac{\frac{A_P}{s}}{1+\frac{A_P}{s}\cdot A_{Tr}\cdot A_P\cdot A_{Ac}}$$

$$G^*(s)=\frac{A_P}{s+A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}=\frac{\frac{A_P}{A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}}{\frac{1}{A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}\cdot s+1}=\frac{\frac{1}{A_{Tr}\cdot A_C\cdot A_{Ac}}}{\frac{1}{A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}\cdot s+1}=\frac{A^*}{T^*\cdot s+1}$$

Free outflow tank

Input flow rate $\Rightarrow$ liquid level:

$$G^*(s)=\frac{h(s)}{W_{in}(s)}=\frac{c(s)}{d(s)}=\frac{G_P(s)}{1+G_P(s)\cdot G_{Tr}(s)\cdot G_C(s)\cdot G_{Ac}(s)}=\frac{\frac{A_P}{T_P\cdot s+1}}{1+\frac{A_P}{T_P\cdot s+1}\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}$$

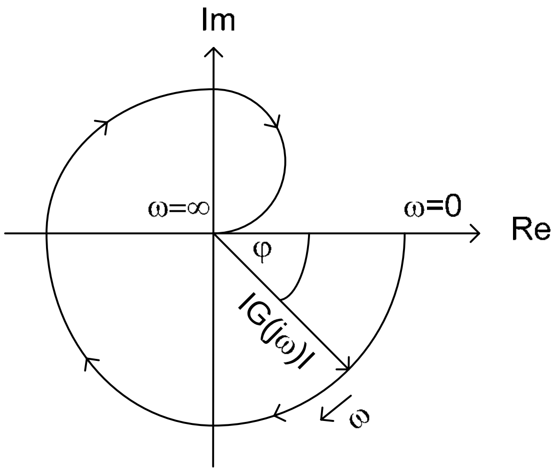
$$G^*(s)=\frac{A_P}{T_P\cdot s+1+A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}=\frac{\frac{A_P}{1+A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}}{\frac{T_P}{1+A_P\cdot A_{Tr}\cdot A_C\cdot A_{Ac}}\cdot s+1}=\frac{A^*}{T^*\cdot s+1}$$

<u>FREQUENCY ANALYSIS</u>

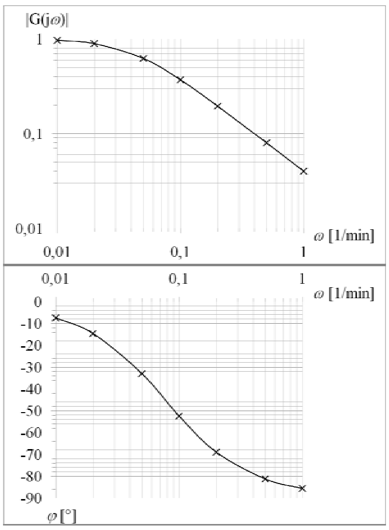
<u>Frequency function from transfer function</u>

$$G(s) \rightarrow G(j\omega)$$

<u>Nyquist diagram</u>



<u>Bode diagram</u>



<u>One point of the frequency function</u>

$$G(j\omega) = \langle \mathrm{Re} \rangle + j \cdot \langle \mathrm{Im} \rangle$$

$$|G(j\omega)| = \sqrt{\langle \mathrm{Re} \rangle^2 + \langle \mathrm{Im} \rangle^2} \qquad \varphi = \arctan \frac{\langle \mathrm{Im} \rangle}{\langle \mathrm{Re} \rangle}$$

<u>Consecutively connected elements:</u>

$$G^*(s) = \prod G_i(s)$$

$$G^*(j\omega) = \prod G_i(j\omega)$$

$$|G^*(j\omega)| = \prod |G_i(j\omega)| \qquad \varphi^* = \sum \varphi_i$$

Frequency functions of basic elements

	$G(s)$	$\lvert G(i\cdot\omega)\rvert$	φ
1st order	$\frac{A}{Ts+1}$	$\frac{\lvert A\rvert}{\sqrt{1+\omega^2T^2}}$	$\arctan(-\omega T)$
2nd order	$\frac{A}{T^2s^2+2\xi Ts+1}$	$\frac{\lvert A\rvert}{\sqrt{(1-\omega^2T^2)+(2\xi T\omega)^2}}$	$\arctan\left(\frac{2\xi\omega T}{1-\omega^2T^2}\right)$
Dead time	$A\cdot e^{-T_D\cdot s}$	A	$-\omega\cdot T_D$
I	$\frac{A_I}{s}=\frac{1}{I\cdot s}$	$\frac{A_I}{\omega}=\frac{1}{I\cdot\omega}$	-90^o
PI	$A_C\cdot\left(1+\frac{1}{I\cdot s}\right)$	$A_C\cdot\sqrt{1+\frac{1}{I^2\cdot\omega^2}}$	
PID	$A_C\cdot\left(1+\frac{1}{I\cdot s}+D\cdot s\right)$		

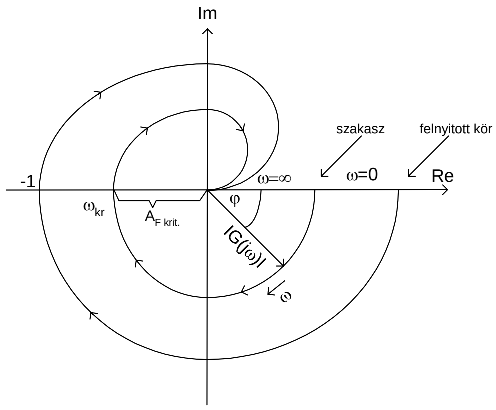
Terms

Controlled section: Everything in the loop without the controller itself

Open loop: Everything in the loop including the controller too

Critical values

Frequency function of the controlled section → 1/critical gain of the section at –180°.
Frequency function of the open loop with P controller and critical gain intersects –1.
Frequency function reaches –180° at ritical frequency.



Tuning

Ziegler-Nichols' suggested tuning
based on closed loop cycling or open loop transfer function

	A_C	I	D
P	$A_C \le 0.5\cdot A_{C,crit}$	∞	0
PI	$A_C \le 0.45\cdot A_{C,crit}$	$I \ge 0.8\cdot T_{crit}$	0
PID	$A_C \le 0.6\cdot A_{C,crit}$	$I \ge 0.5\cdot T_{crit}$	$D < 0.125\cdot T_{crit}$

Margins

Phase margin: phase lag of the open loop at amplitude ratio 1 is how much less then (the critical) 180°.

Gain margin: amplitude ratio of the open loop at (the critical) phase lag 180°.