CONTROL CHARTS

- variables
- attributes

common (chance) cause	left to chance
specific (assignable) cause	identified and eliminated

Attribute control charts

- charts for defectives (*np* and *p*)

  based on Binomial distribution

- charts for occurrences (defects) (*c* and *u*)

  based on Poisson distribution

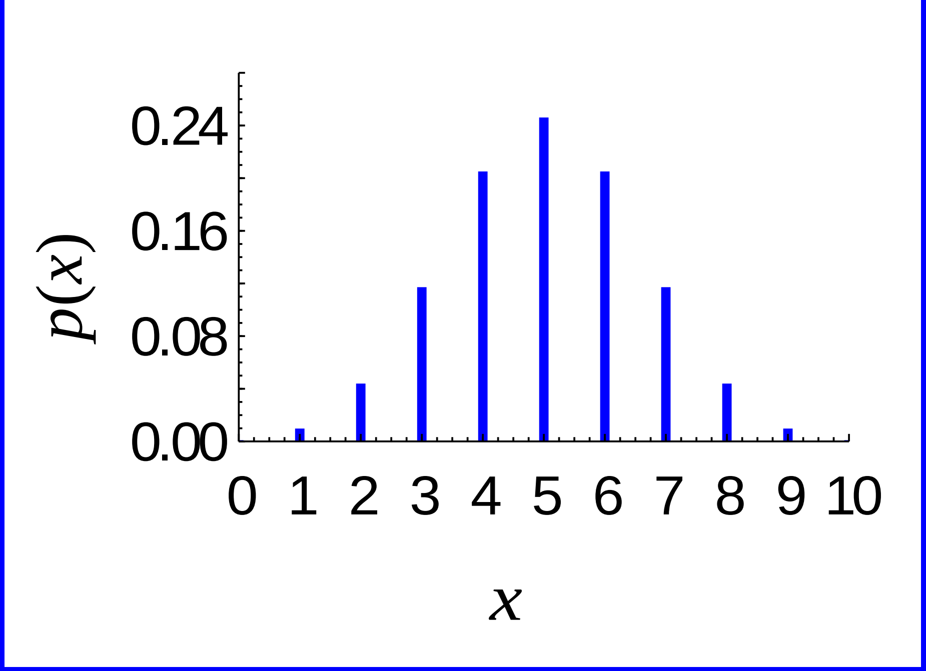
Control charts for count of defectives: *np chart*

p is the proportion of defectives in the population (process), its estimate is the proportion of defectives in the sample:

$$\hat{p} = \frac{x}{n}$$

x is the number of defectives in the sample of size *n*

Binomial distribution:



$$p(x)=\binom{n}{x}p^{x}(1-p)^{n-x}$$

$$\mu_x = E(x) = np$$

$$\sigma_x^2 = Var(x) = np(1-p)$$

$$\mu_{x/n} = E\left(\frac{x}{n}\right) = p$$

$$\sigma_{x/n}^2 = Var\left(\frac{x}{n}\right) = \frac{p(1-p)}{n}$$

Criteria for application:

• The elements may take two kinds of values (dichotomous) e.g. *”yes/no”*.

Probability of the *”yes”* event is p

[(1-p) is for the complementary *no”* event],

x is the # success from n experiment.

- The *i*-th element has the same chance for *"yes"* as has the *i*+1-th

Imagine taking elements from a lot of 20 (10% is non-conforming)

Chance of the first element for being non-conforming:

If it is non-conforming, chance for the second element:

Replacement would be a solution.

- Binomial distribution for sampling without replacement is justified if $n \ll N$

The parameters of the *np* chart according to the ±3σ rule

$$E(x) = np$$

$$CL_{np} = n\bar{p}$$

$$Var(x) = np(1-p)$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

If *LCL* is <0, set to zero.

$\bar{p}$ is the average proportion of defectives

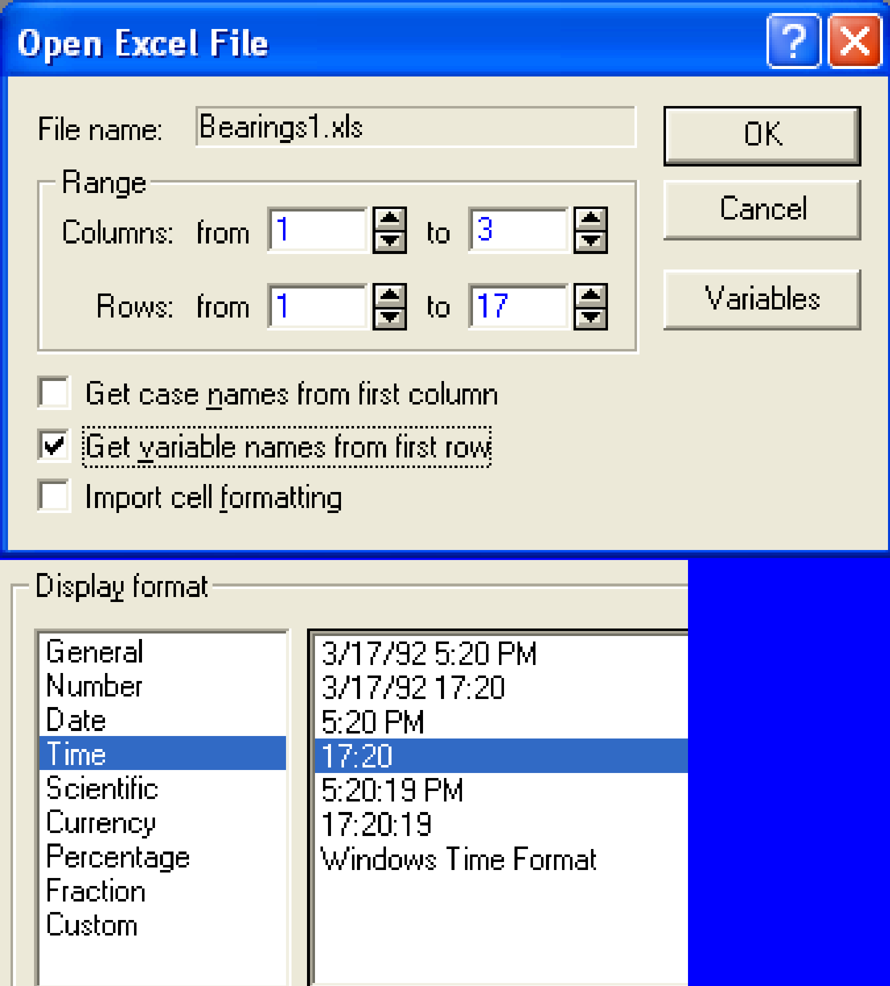
Example 16

50 pieces are drawn in each half an hour from a process producing …. of defectives:

time	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30
D (*np*)	0	5	3	7	5	5	4	8

time	12:00	12:30	13:00	13:30	14:00	14:30	15:00	15:30
D (*np*)	0	5	3	7	5	5	4	8

Prepare an *np* chart assuming the situation of a Phase I study! Bearings1.xls

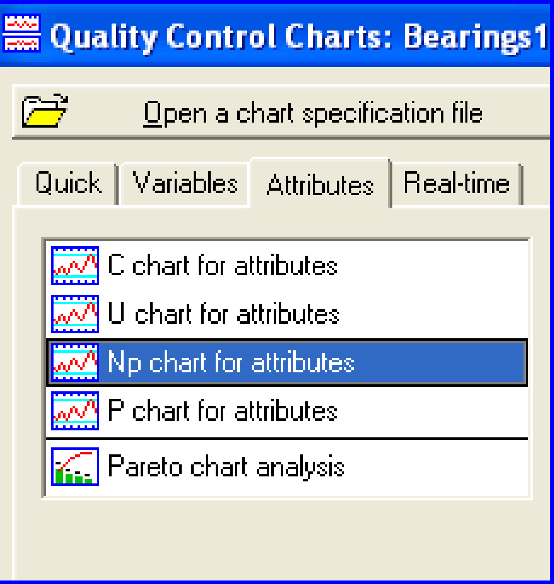


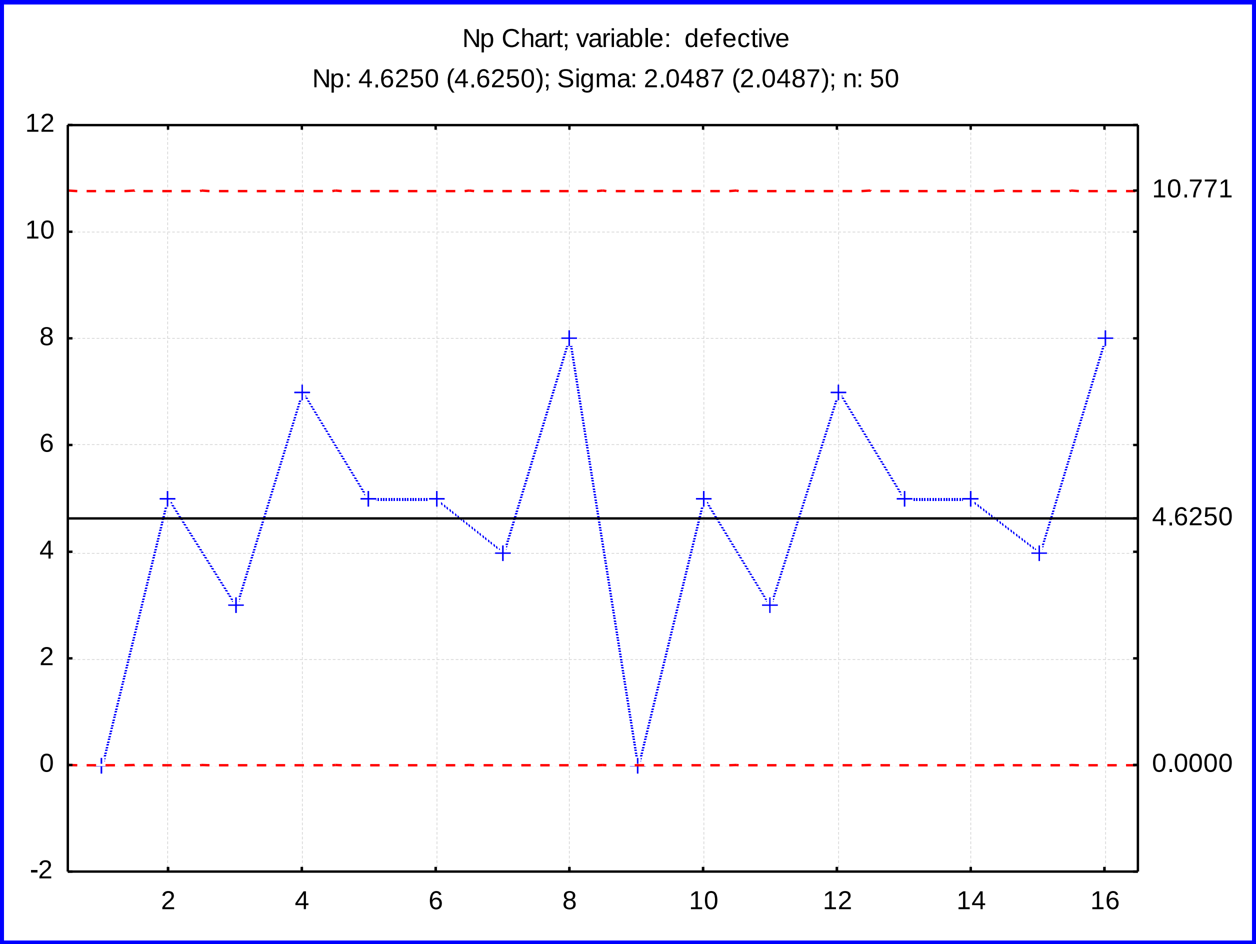
	Sheet1		
	1 time	2 defective	3 N
1	8:00	0	50
2	8:30	5	50
3	9:00	3	50
4	9:30	7	50

open an Excel file

	Sheet1		
	1 time	2 defective	3 N
1	0.333333	0	50
2	0.354167	5	50
3	0.375	3	50
4	0.395833	7	50
5	0.416667	5	50
6	0.4375	5	50
7	0.458333	4	50
8	0.479167	8	50
9	0.5	0	50
10	0.520833	5	50
11	0.541667	3	50
12	0.5625	7	50
13	0.583333	5	50
14	0.604167	5	50
15	0.625	4	50
16	0.645833	8	50

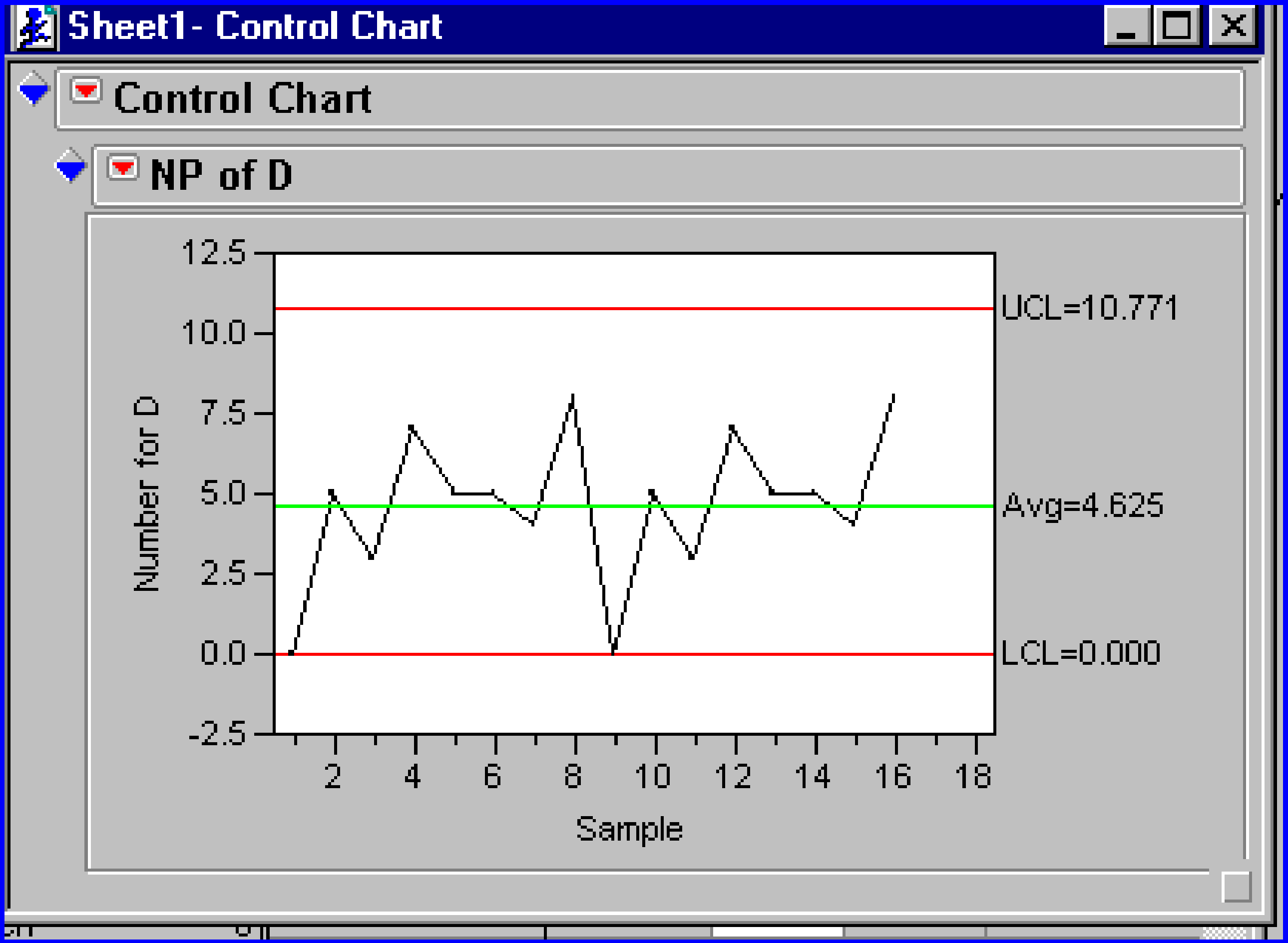
Statistics>Industrial Statistics>Quality Control Charts
Np chart for attributes
Counts: Defective, Sample size: N



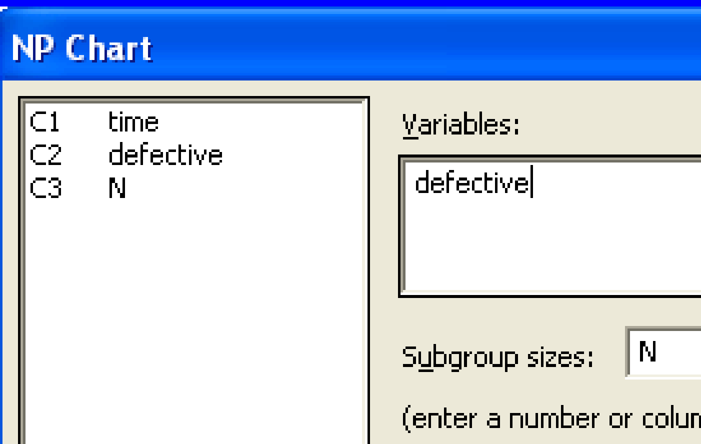


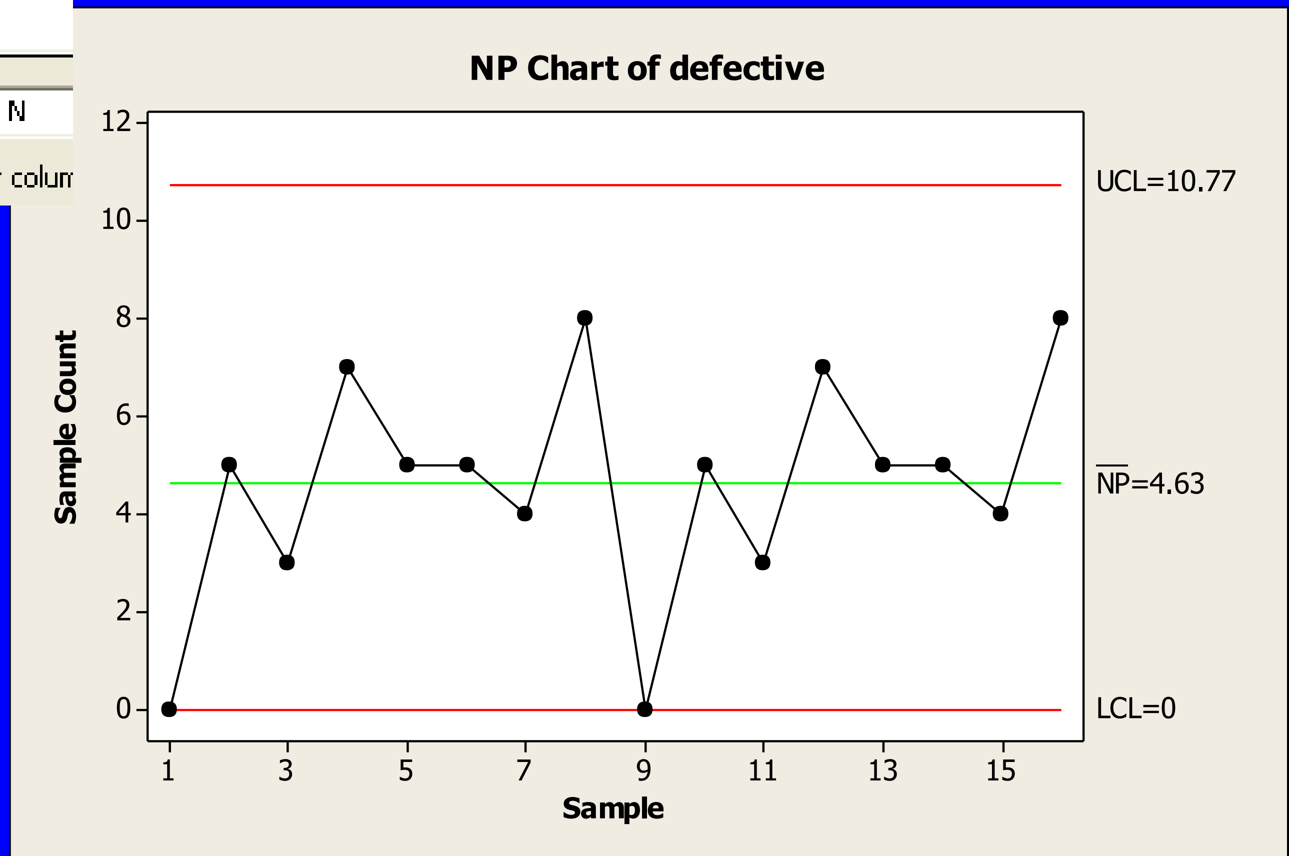
Why do we have a single chart?

Open Data Table: Bearings1.xls
Graph>ControlChart
Chart Type: np; Process: D; Sample Size: N



Minitab>Stat>Control Charts> Attributes Charts >NP





Example 21

Determine the minimum required sample size for obtaining a non-zero LCL if p=0.03!

$$LCL_{np} = np - 3\sqrt{np(1-p)} > 0$$

$$n > \frac{9(1-p)}{p}$$

Example 22

Determine the minimum required sample size for obtaining at least one defective at 99% probability, that is $P(D>0)\geq 0.99$, if $p=0.03$!

$$P\left(D>0\right)=1-P\left(D=0\right)=$$

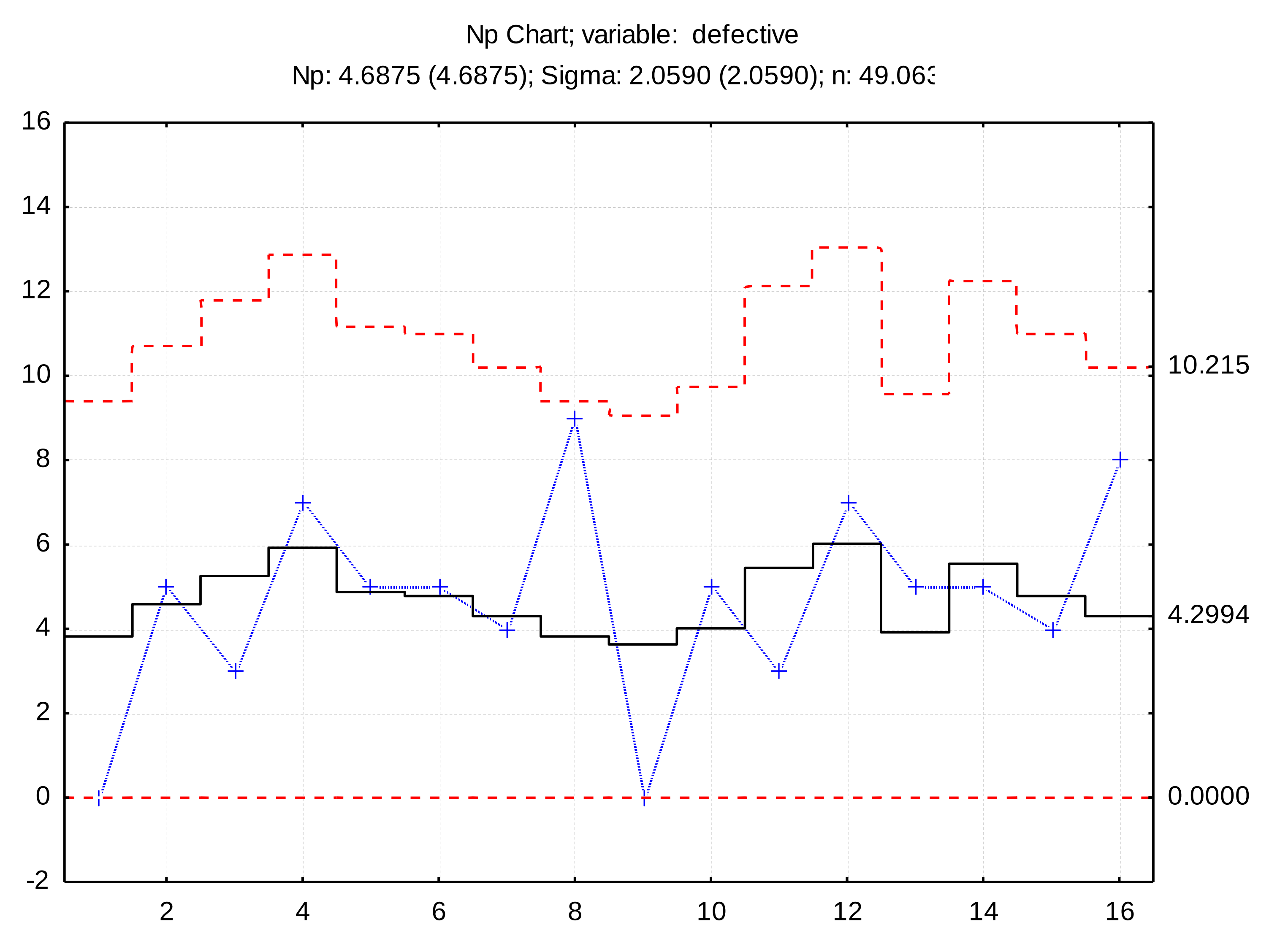
Case of not constant sample size

$$CL_{np} = n\bar{p}$$

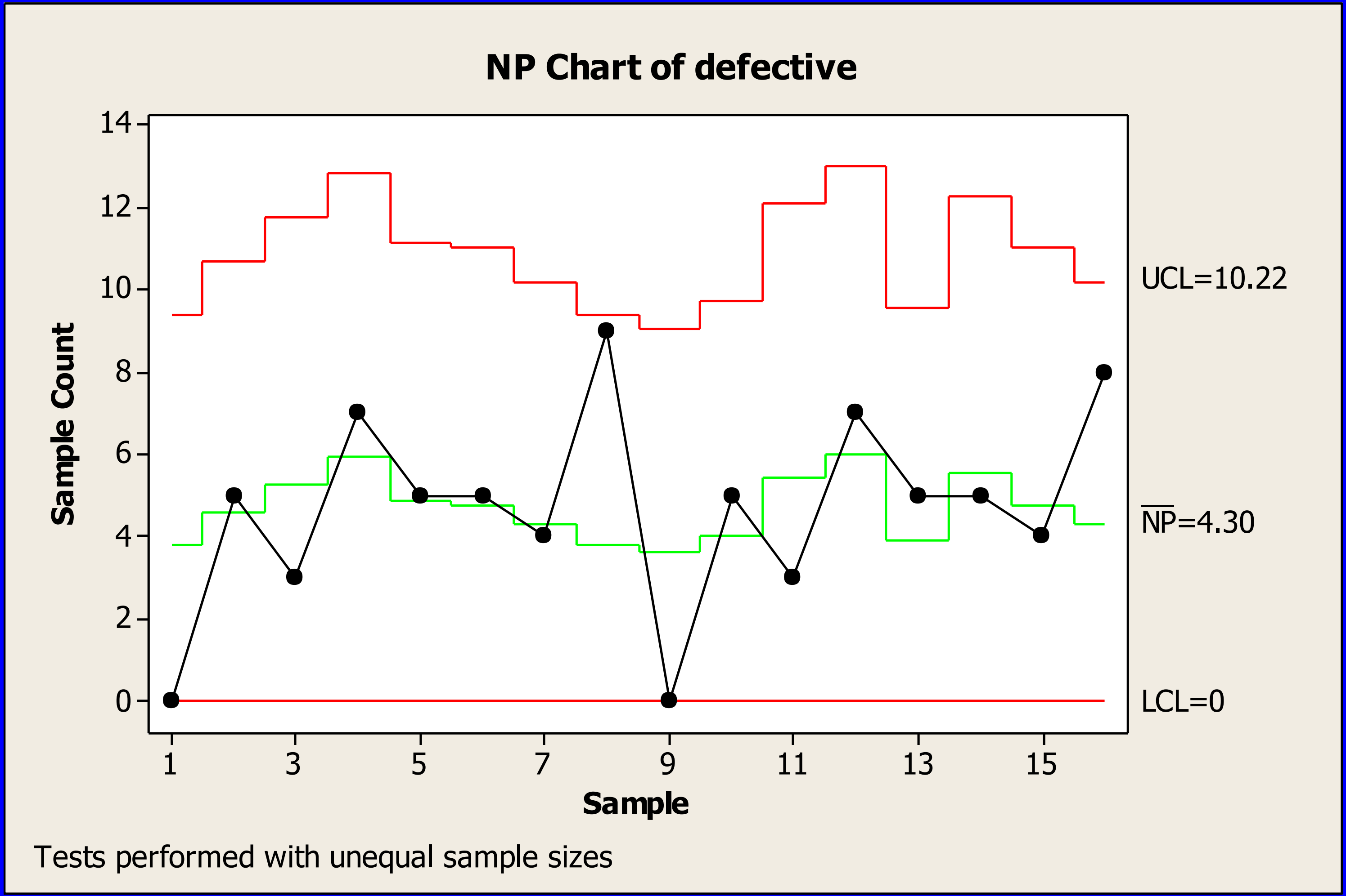
$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}\left(1 - \bar{p}\right)}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}\left(1 - \bar{p}\right)}$$

np- chart with changing sample size



np-chart with changing sample size



Control chart for proportion of defectives: *p chart*

$$\hat{p} = \frac{D}{n} \qquad E(\hat{p}) = p \qquad Var(\hat{p}) = \frac{p(1-p)}{n}$$

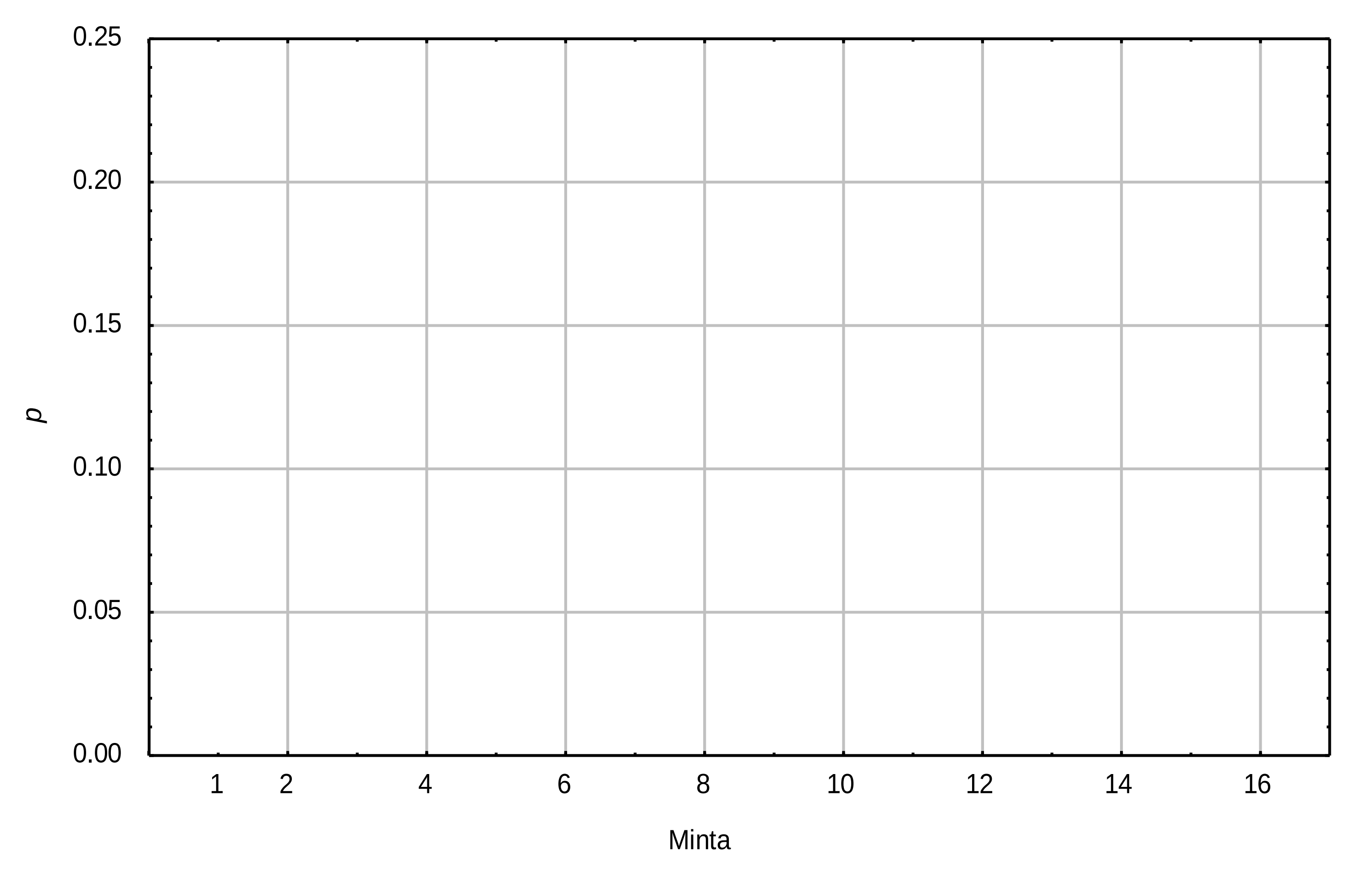
The parameters according to the ±3σ rule:

$$CL_p = \overline{p}$$

$$UCL_p = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \qquad LCL_p = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Example 23

Prepare a *p* chart for data given in Example 20!



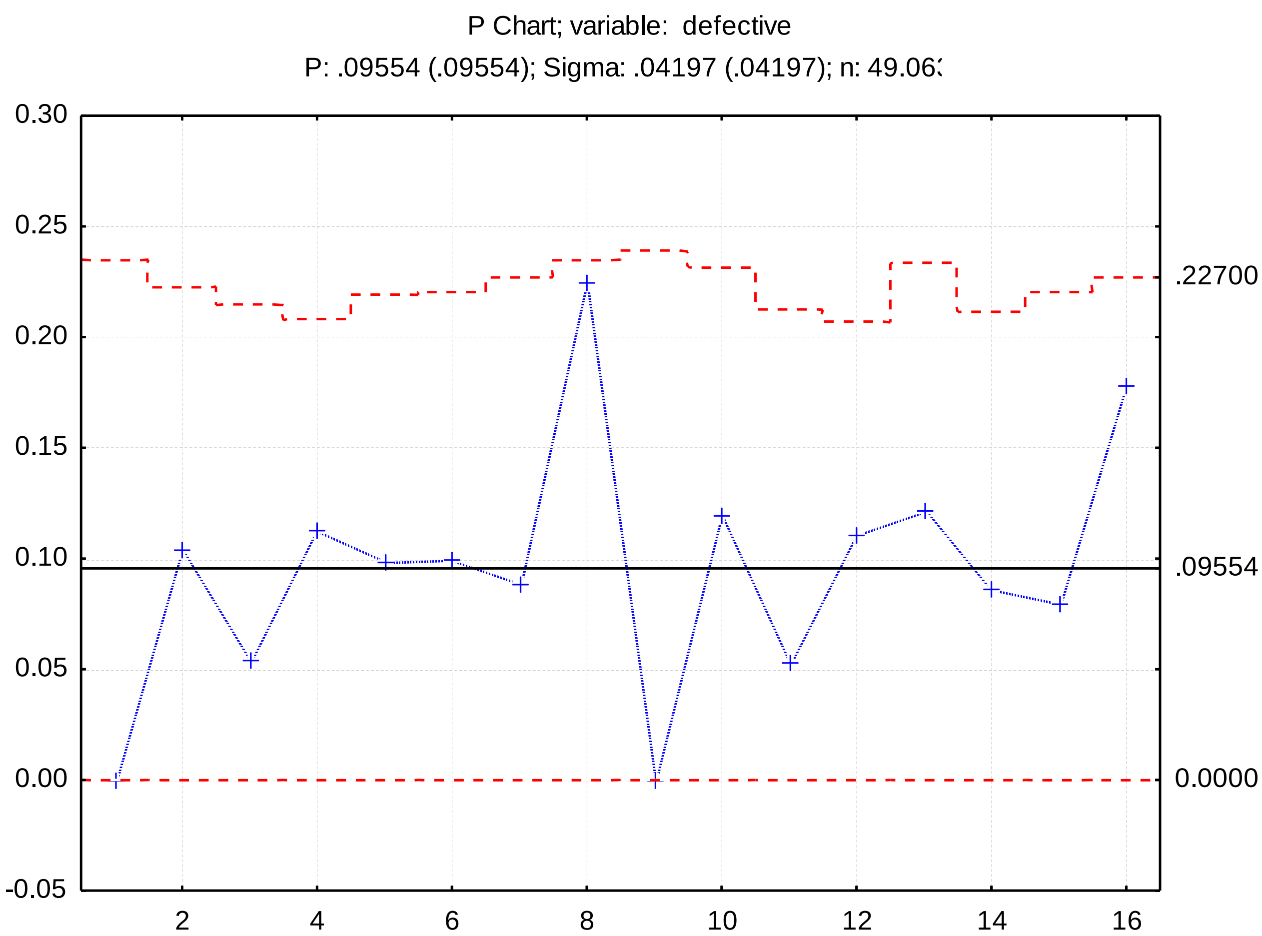
time	D	n
8:00	0	40
8:30	5	48
9:00	3	55
9:30	7	62
10:00	5	51
10:30	5	50
11:00	4	45
11:30	9	40
12:00	0	38
12:30	5	42
13:00	3	57
13:30	7	63
14:00	5	41
14:30	5	58
15:00	4	50
15:30	8	45

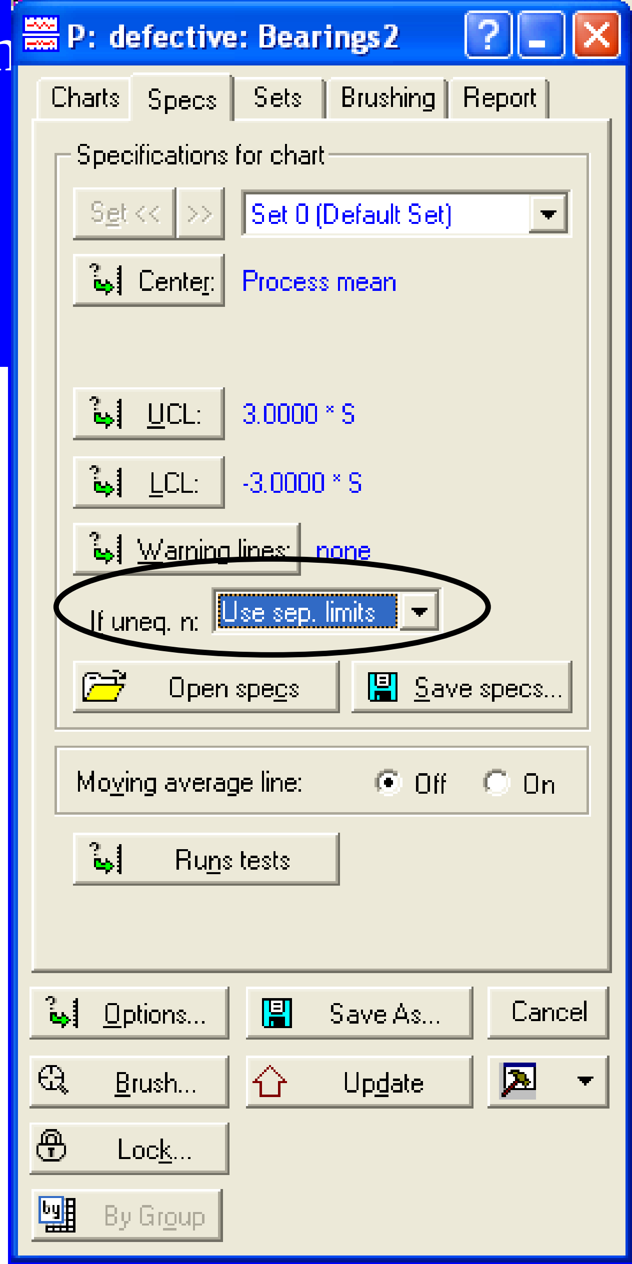
Example 24

Prepare a p chart assuming the situation of a Phase I study! (Bearings2.xls)

Statistics>Industrial Statistics>Quality Control Ch
p chart for attributes
Counts: Defective, Sample size: N

using actual sizes of subgroups



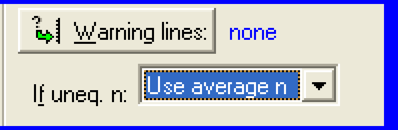


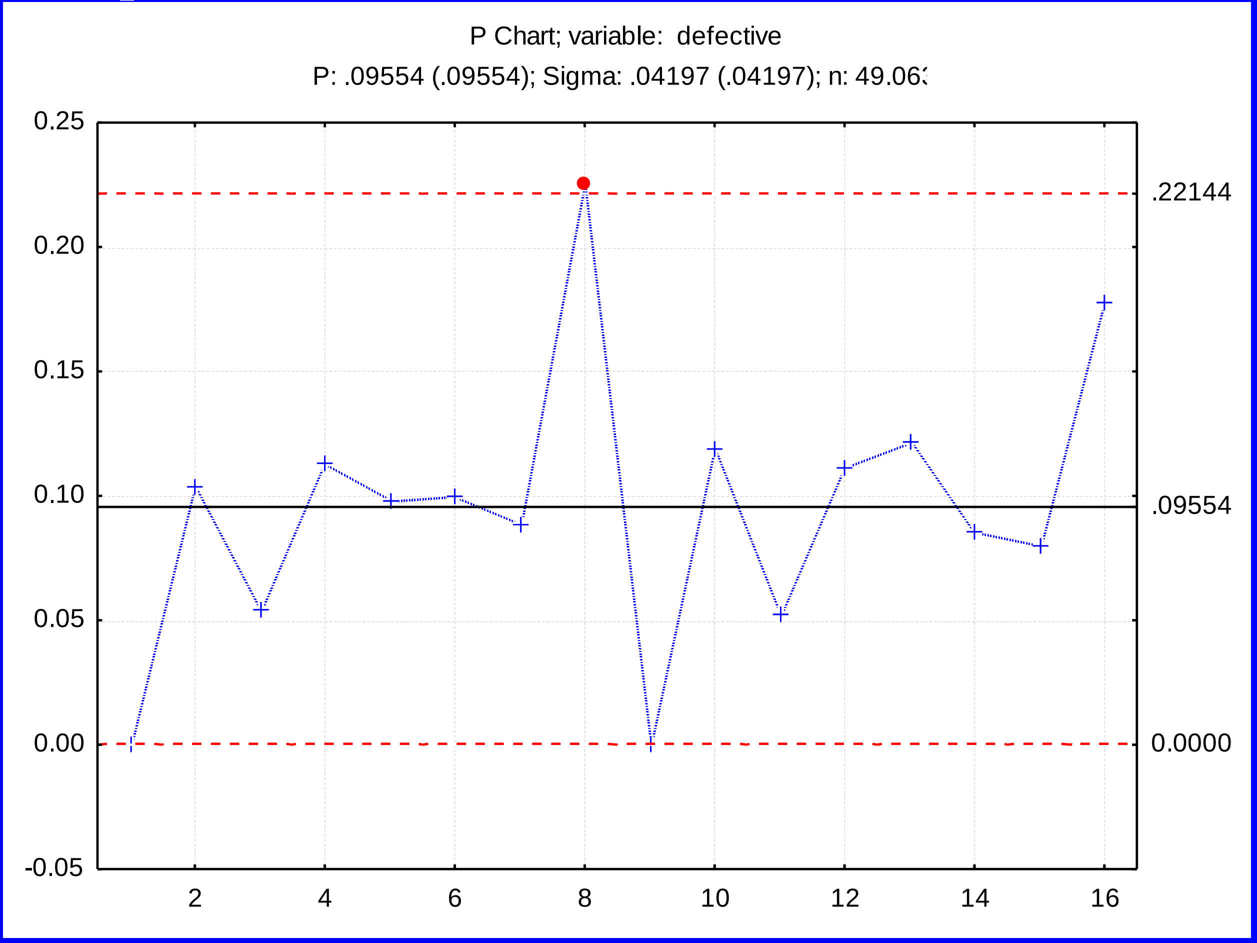
p chart with average control limits

Statistics>Industrial Statistics>Quality Control Charts

p chart for attributes

Counts: Defective, Sample size: N

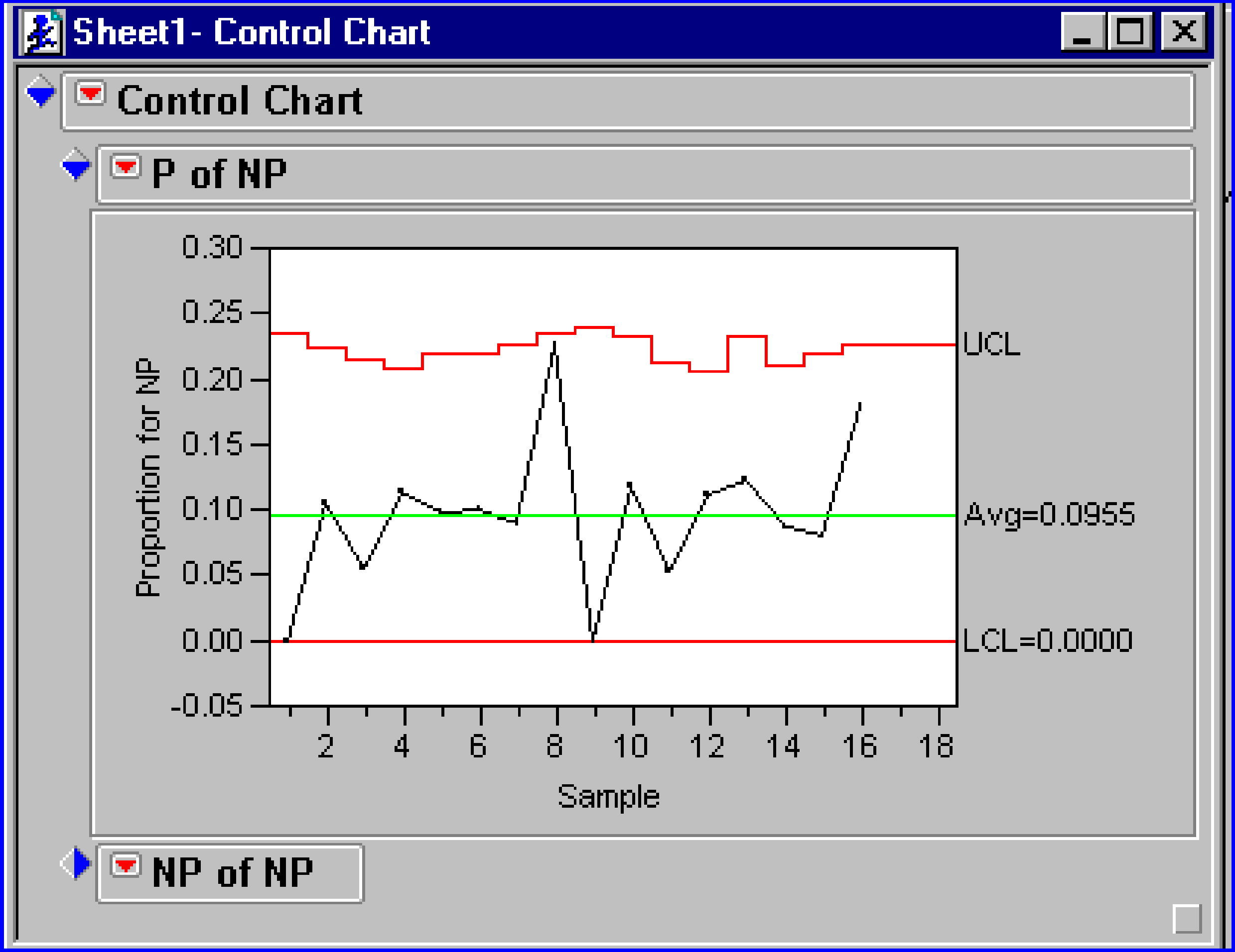




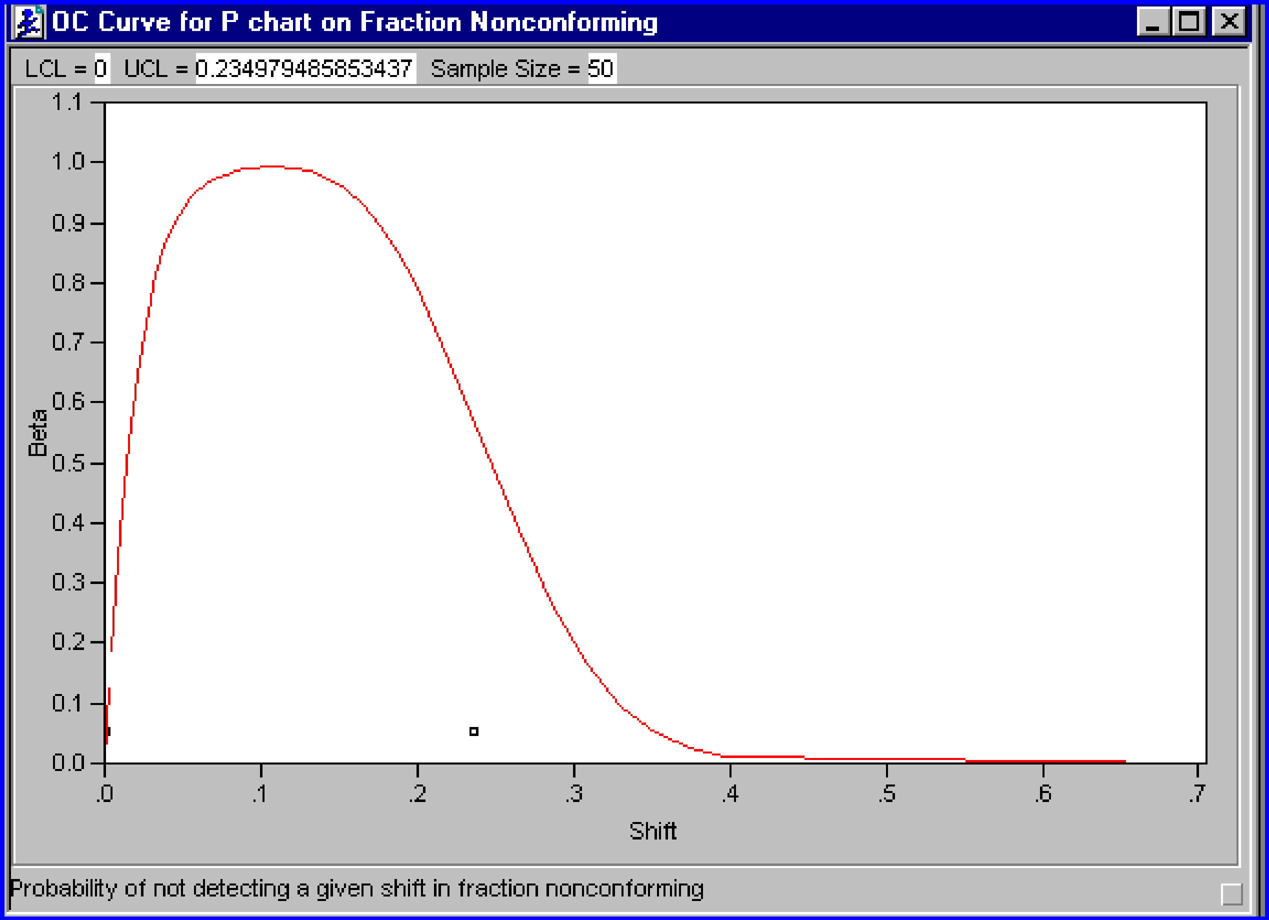
Open Data Table: Bearings2.xls

Graph>ControlChart

Chart Type: p; Process: NP; Sample Size: N



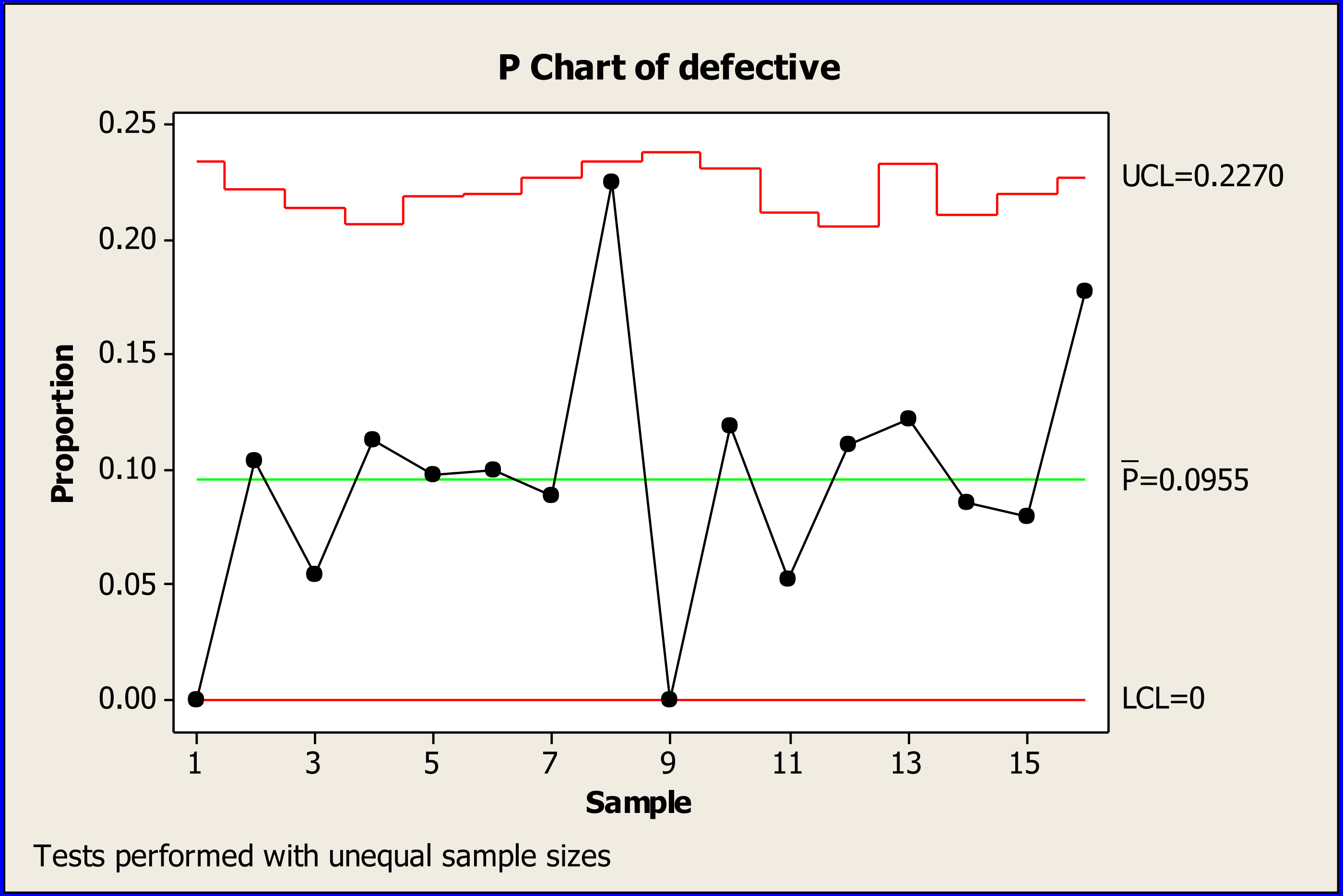
OC curve



OC Curve for P chart on Fraction Nonconforming
LCL = 0 UCL = 0.234979485853437 Sample Size = 50
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Beta
.0
.1
.2
.3
.4
.5
.6
.7
Shift
Probability of not detecting a given shift in fraction nonconforming

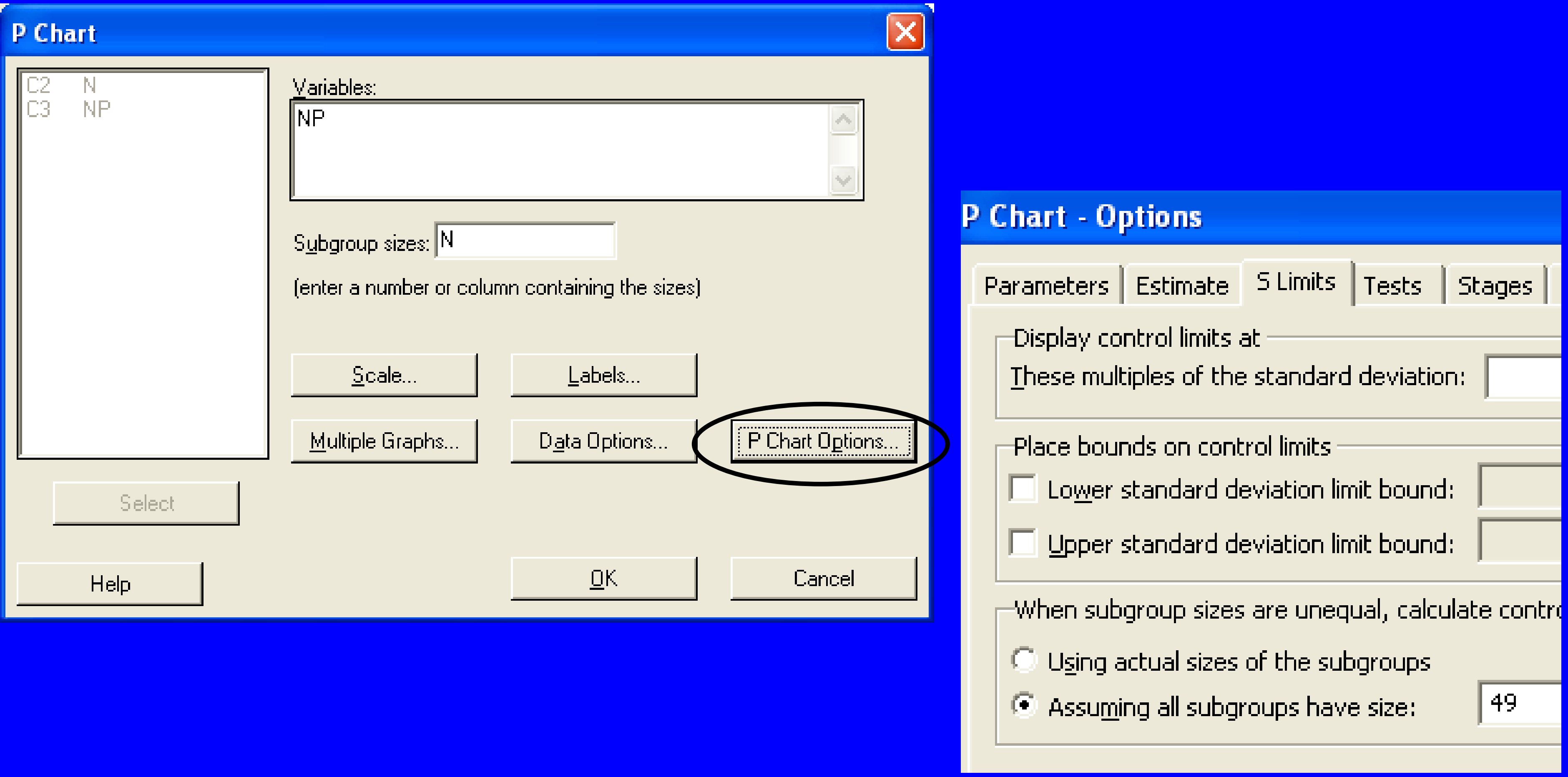
Minitab>Stat>Control Charts>Attribute Charts>P

using actual sizes of subgroups

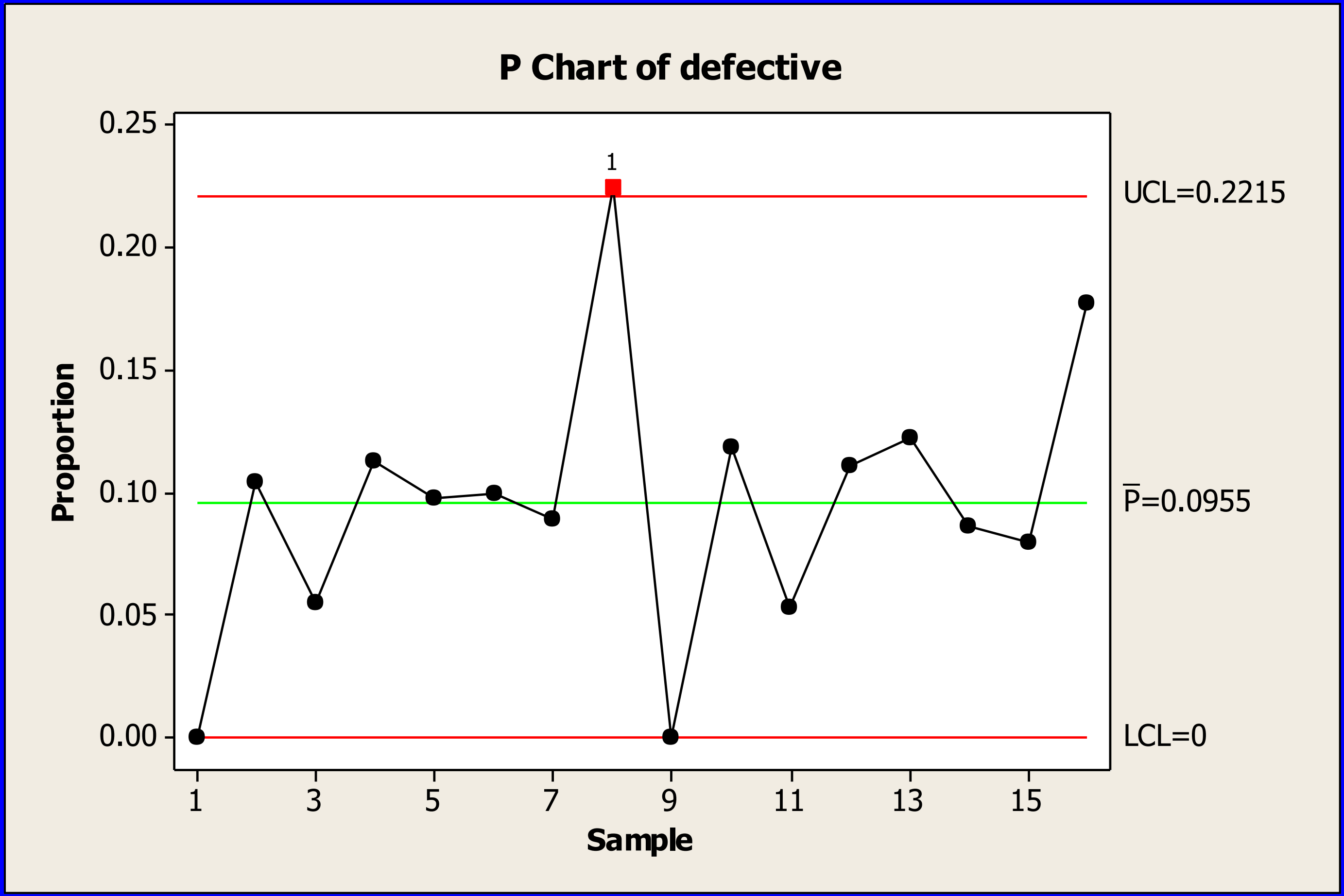


Minitab>Stat>Control Charts>Attribute Charts>P

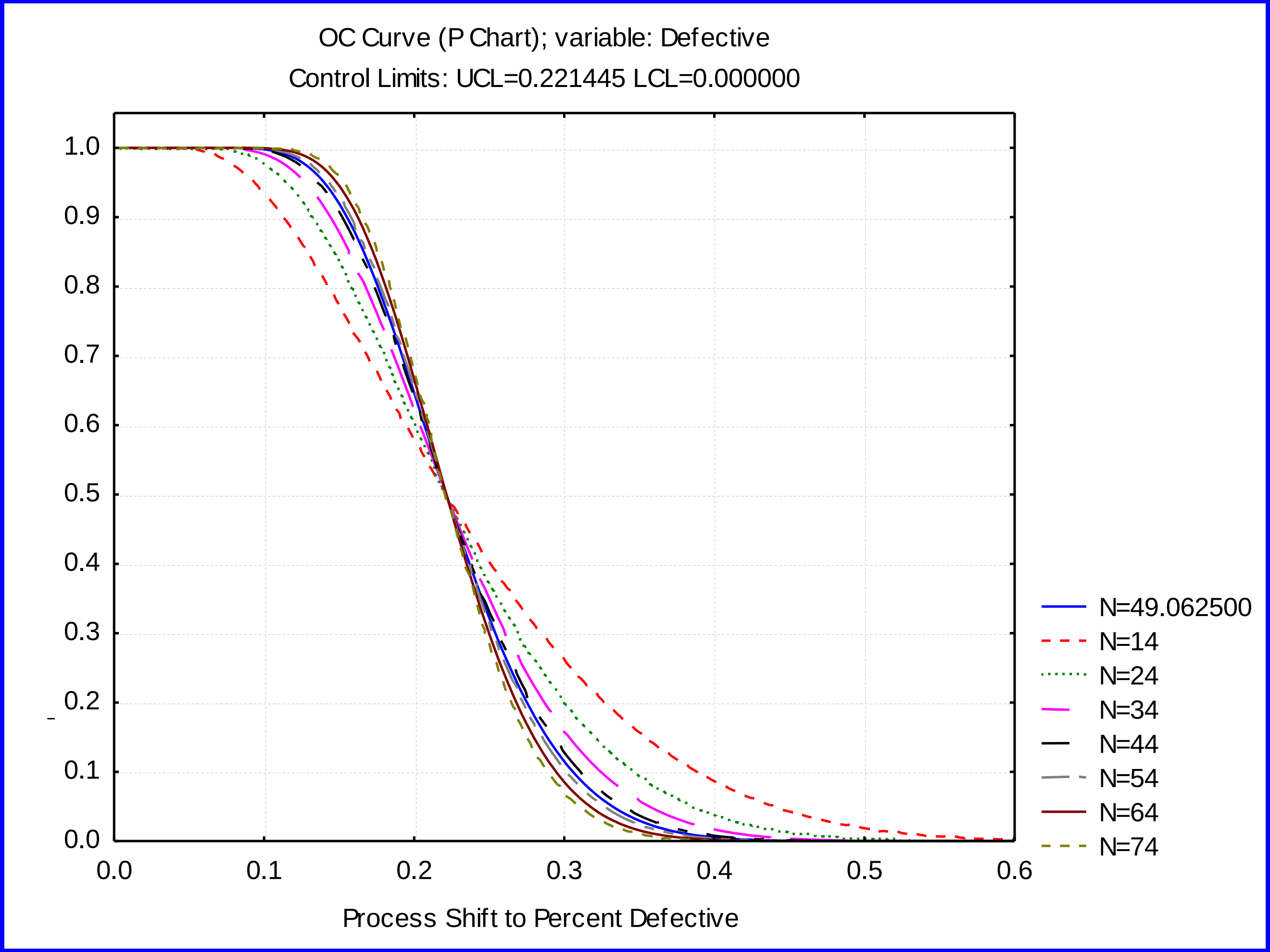
p chart with average control limits



assuming all subgroups have size 49



OC curve


OC Curve (P Chart); variable: Defective
Control Limits: UCL=0.221445 LCL=0.000000
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Process Shift to Percent Defective
N=49.062500
N=14
N=24
N=34
N=44
N=54
N=64
N=74

Control charts for occurrence of defects: *c* chart

Poisson distribution

for modelling rare events

x is the number of occurrences, „from among how many" is not defined

$$p(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Expected value and variance: $E(x) = Var(x) = \lambda$

λ is the expected number of occurrences in a unit

Conditions for applying the Poisson distribution

- occurrence of an event in any unit is independent from than in any other unit
- probability of occurrence of an event in any unit is the same in all units and proportional to the size of the units
- probability of double or multiple occurrences goes to zero by reducing the size of the unit

Defect charts: c chart

$$p(k) = \frac{\lambda^x e^{-\lambda}}{k!}$$

$$\lambda = np$$

$$E(k) = \lambda$$

$$Var(k) = \lambda$$

The *k* average number of defects obtained in Phase I is the estimate of the λ parameter :

$$\bar{c} = \frac{\sum_i^m c_i}{m}$$

c_i # of defects found in sample *i*
m # of samples checked

In Phase II (on-going control) the parameters of the charts using the ±3σ rule:

$$CL_c = \bar{c}$$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

$\bar{c}$ is the value obtained in Phase I.

sample	# defects
1	17
2	14
3	15
4	13
5	7
6	12
7	17
8	12
9	16
10	2

Example 25

The average number of painting defects on car doors manufactured is 2. The doors are sampled for checking, 6 doors are considered as 1 sample.

door2.mtw

Is it a Phase I or Phase II study?

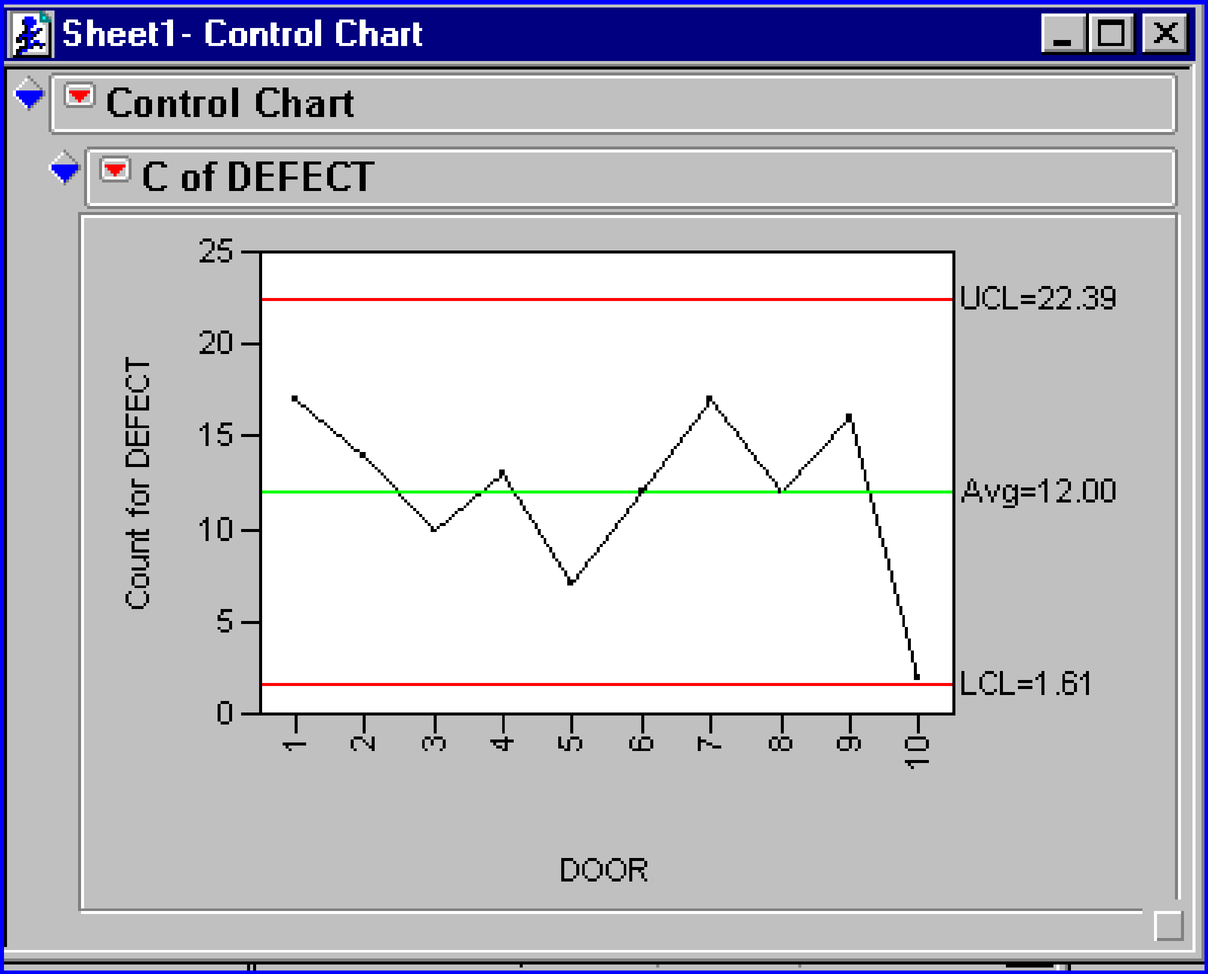
Prepare a *c* chart for checking stability of the process!

Open Data Table: Door.xls

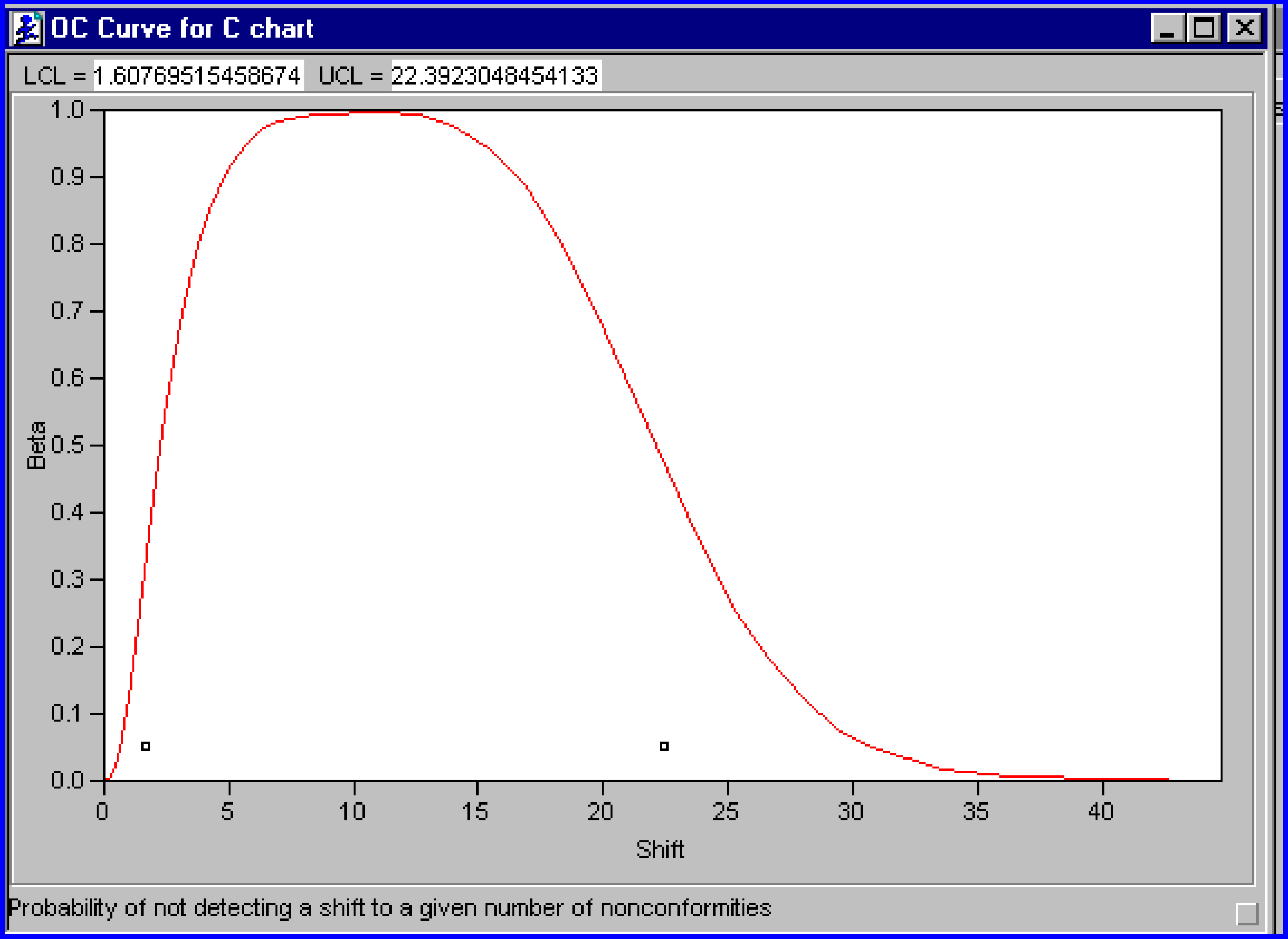
Graph>ControlChart

Chart Type: c; Process: DEFECT;

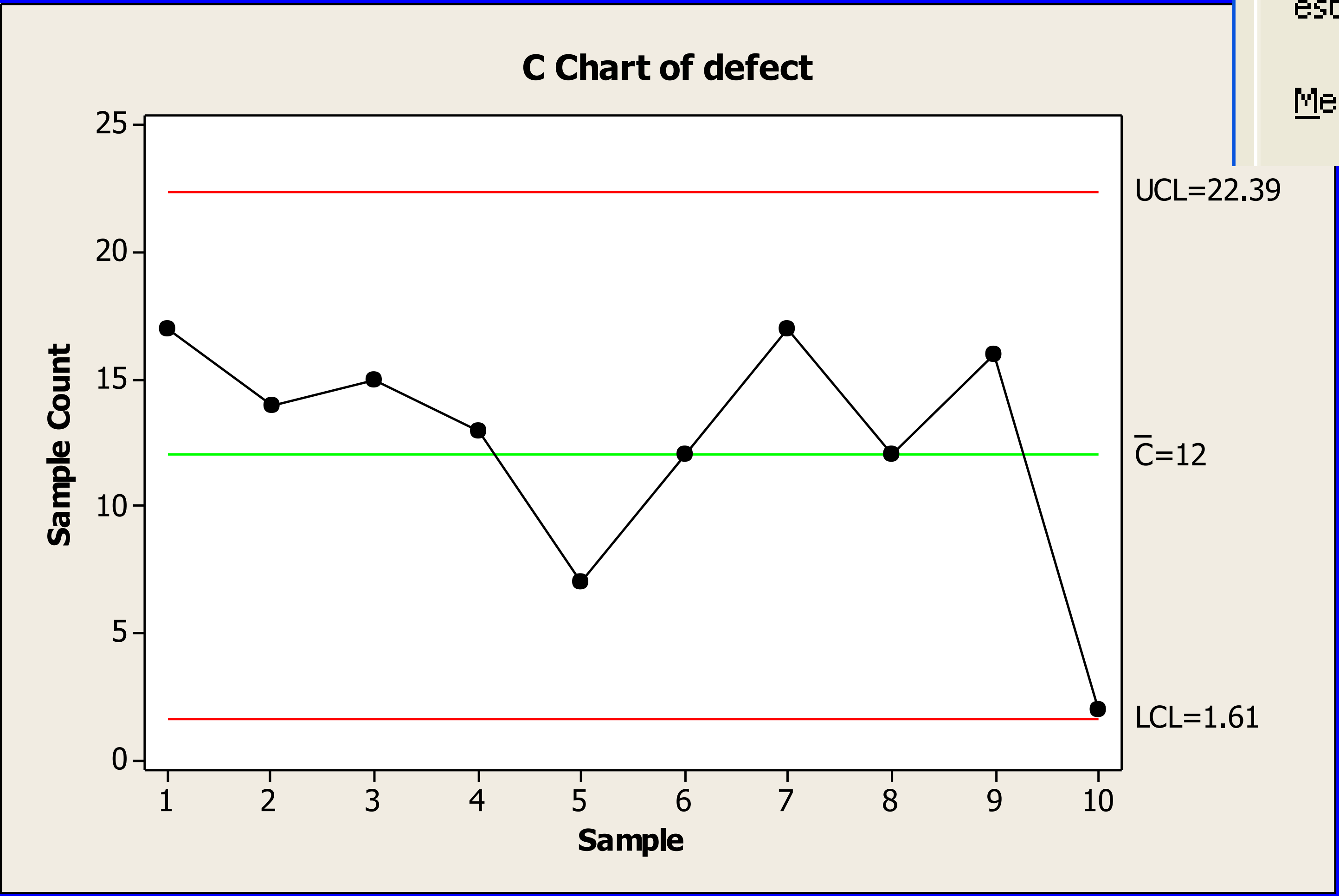
Sample Label: Door

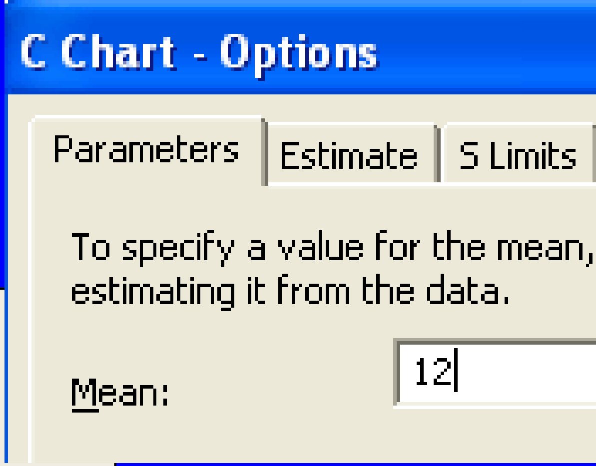


OC curve


OC Curve for C chart
LCL = 1.60769515458674 UCL = 22.3923048454133
Beta
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
35
40
Shift
Probability of not detecting a shift to a given number of nonconformities

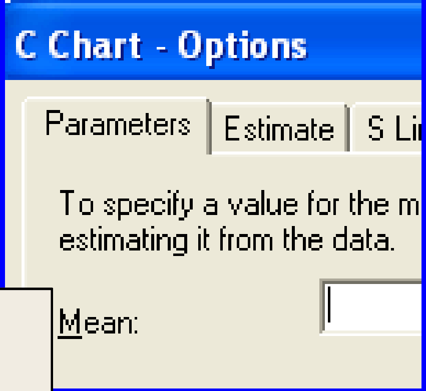
Stat>Control Charts>Attribute Charts>C

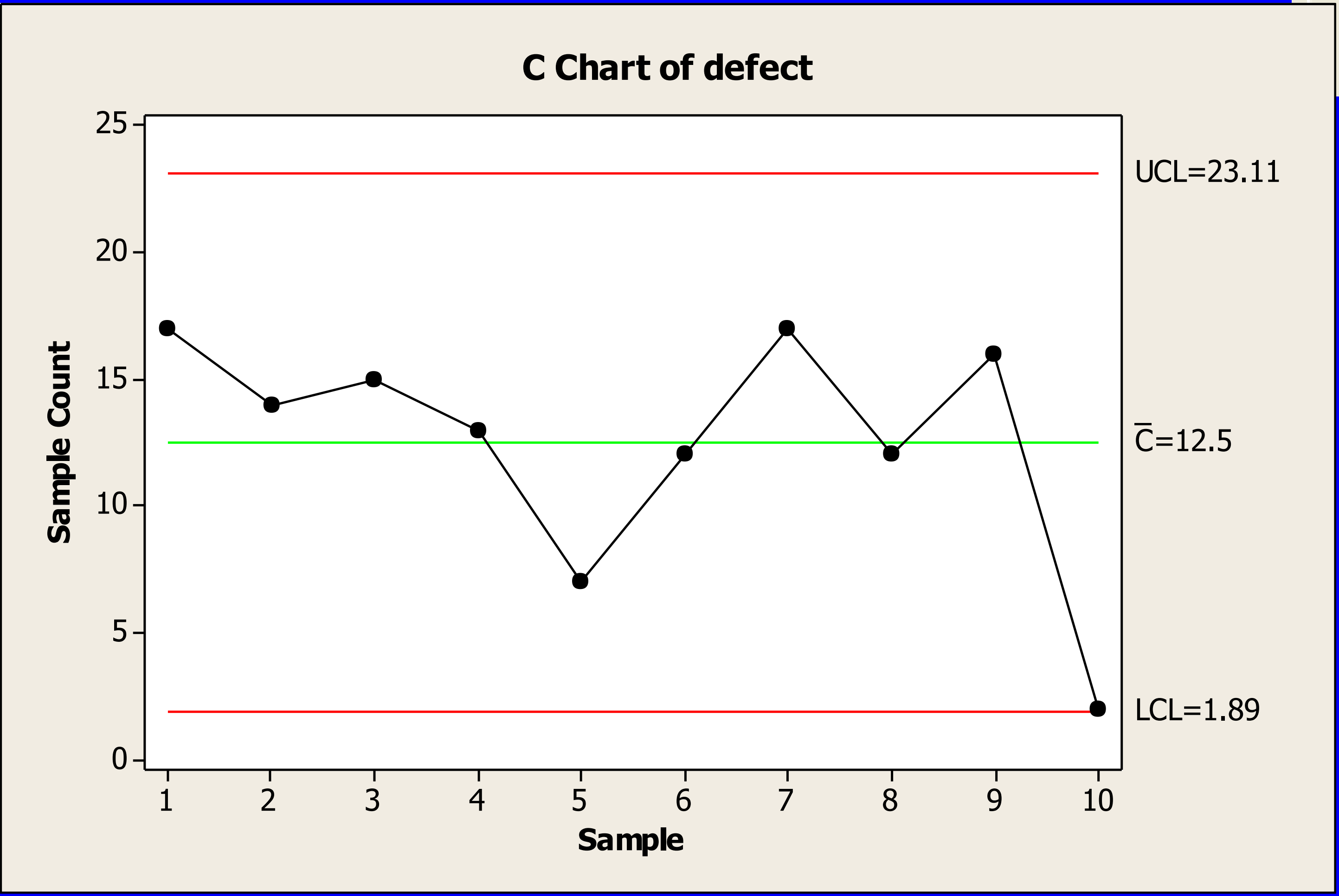




(door2.mtw)

Considering as Phase I study:





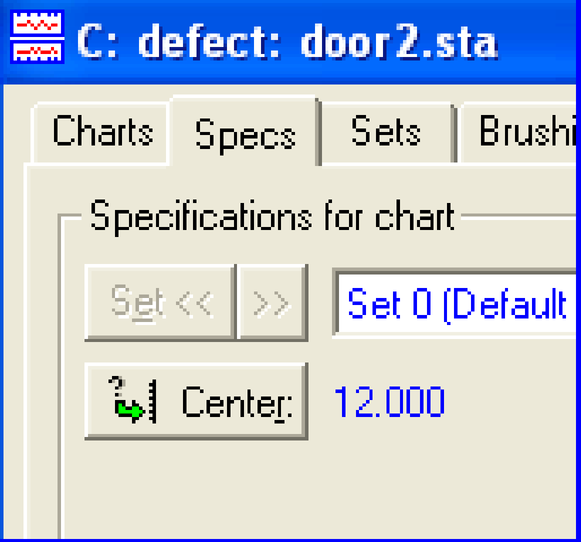
sample	# defects
1	17
2	14
3	15
4	13
5	7
6	12
7	17
8	12
9	16
10	2

Example 20

The average number of painting defects on car doors manufactured is 2. The doors are sampled for checking, 6 doors are considered as a sample.
Prepare a *c* chart for checking stability of the process!

door2.sta

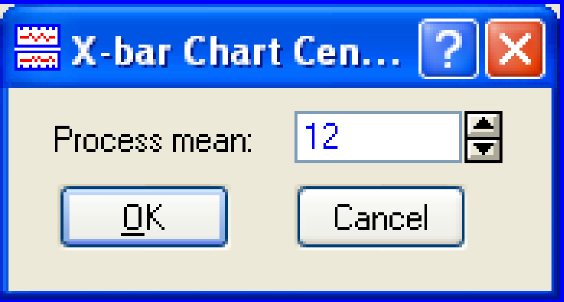
Phase I or Phase II?

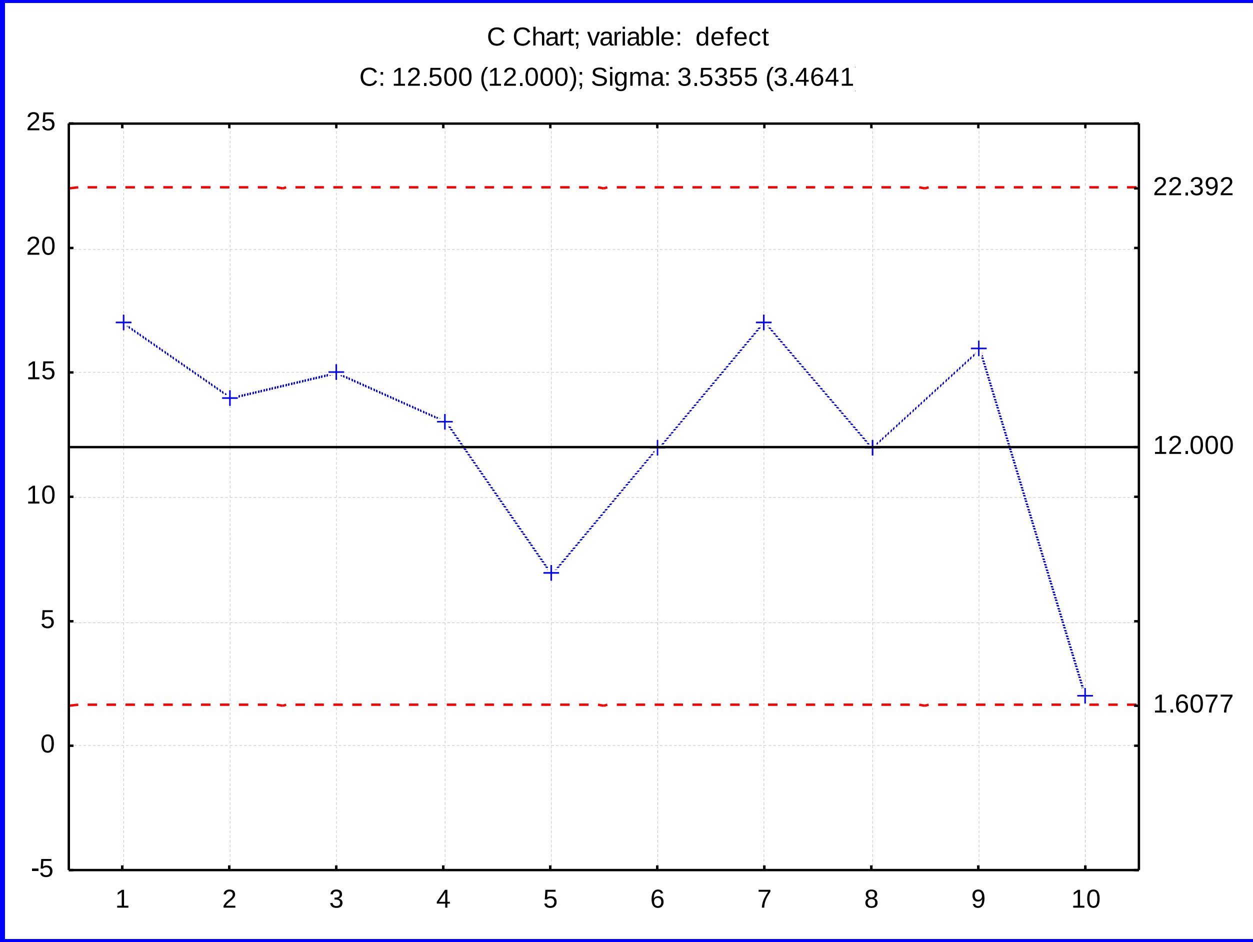


Statistics>Industrial Statistics>Quality Control Charts

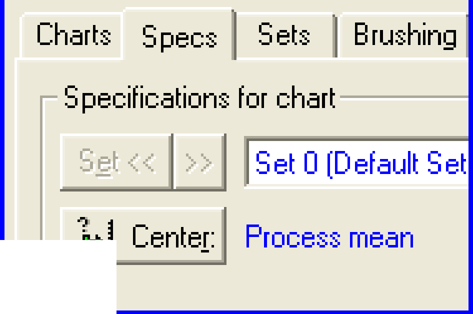
c chart for attributes

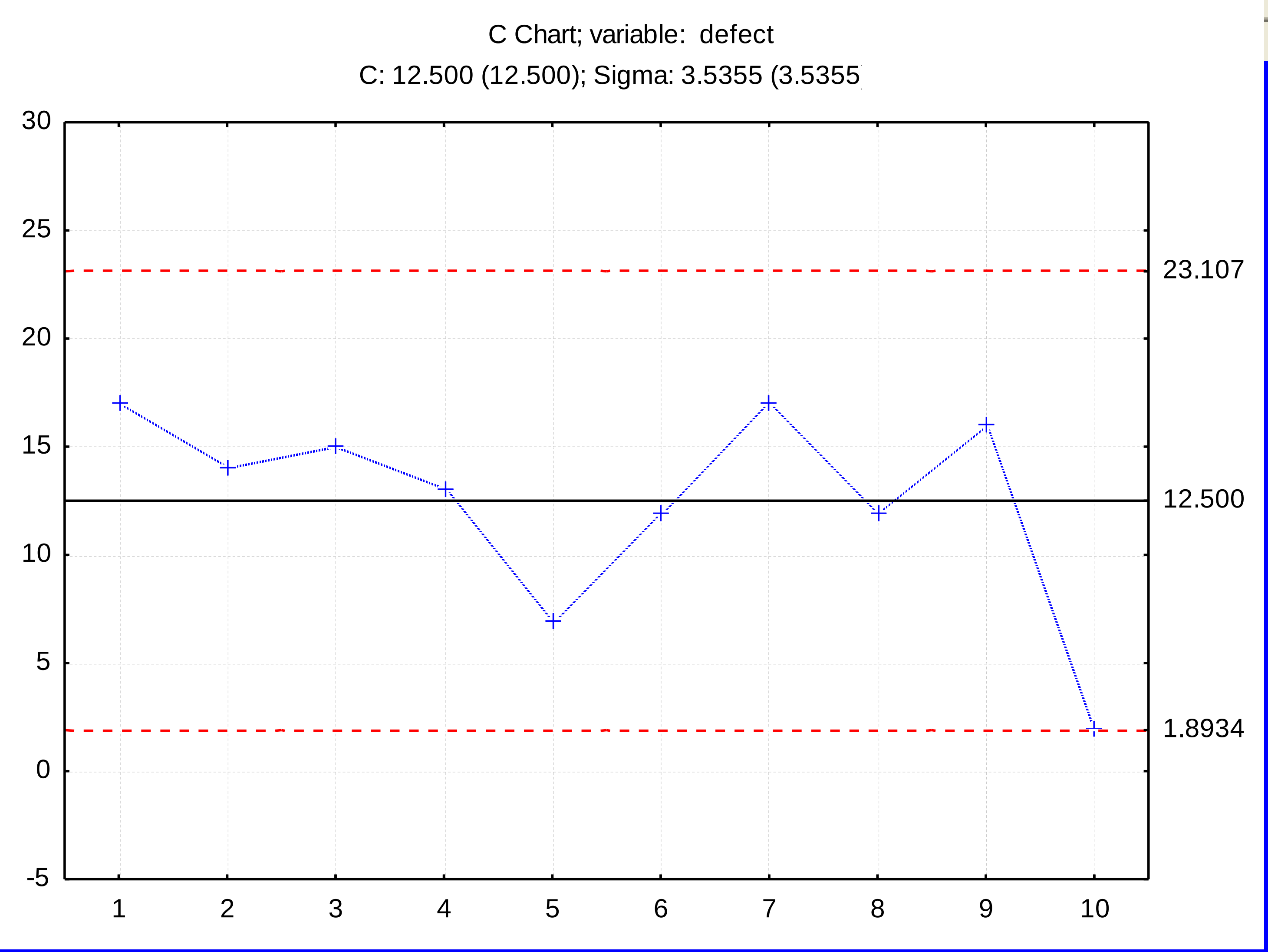
Counts: Defects



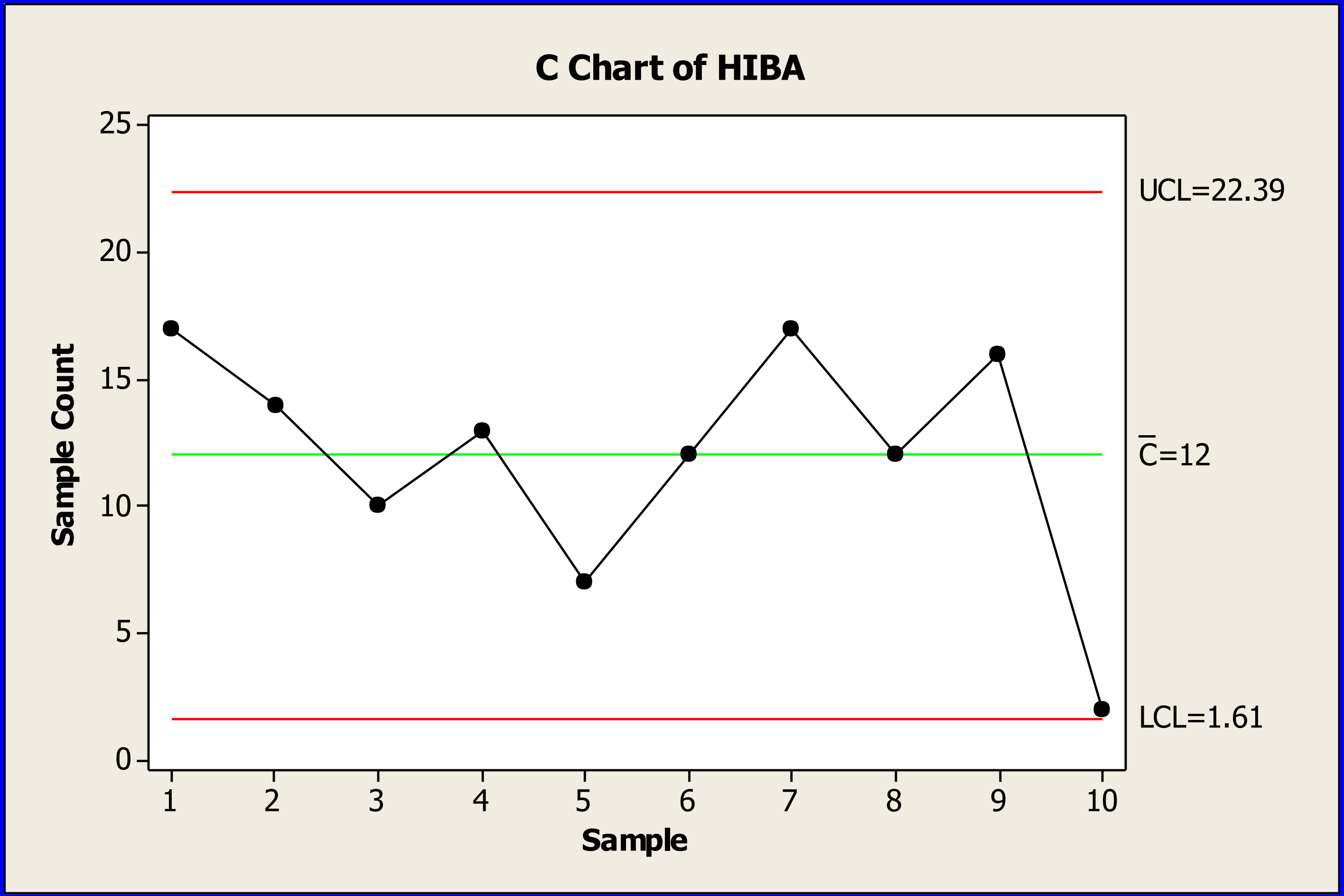


Considering as Phase I study:





Stat>Control Charts>Attribute Charts>c



(ajto.mtw)

Example 26

The average number of painting defects on car doors manufactured is C.
How many (r) doors should be contained in a sample in order to obtain positive LCL?

The average number of defect per sample is $c=Cr$

$$LCL_c = Cr - 3\sqrt{Cr} > 0 \qquad r > \frac{9}{C}$$

E.g. C=2, r>4.5, if r = 5 $\quad LCL_c = 2\cdot 5 - 3\sqrt{2\cdot 5} = 10 - 9.487$

if r = 6 $\quad LCL_c = 2\cdot 6 - 3\sqrt{2\cdot 6} = 12 - 10.392$

Example 18

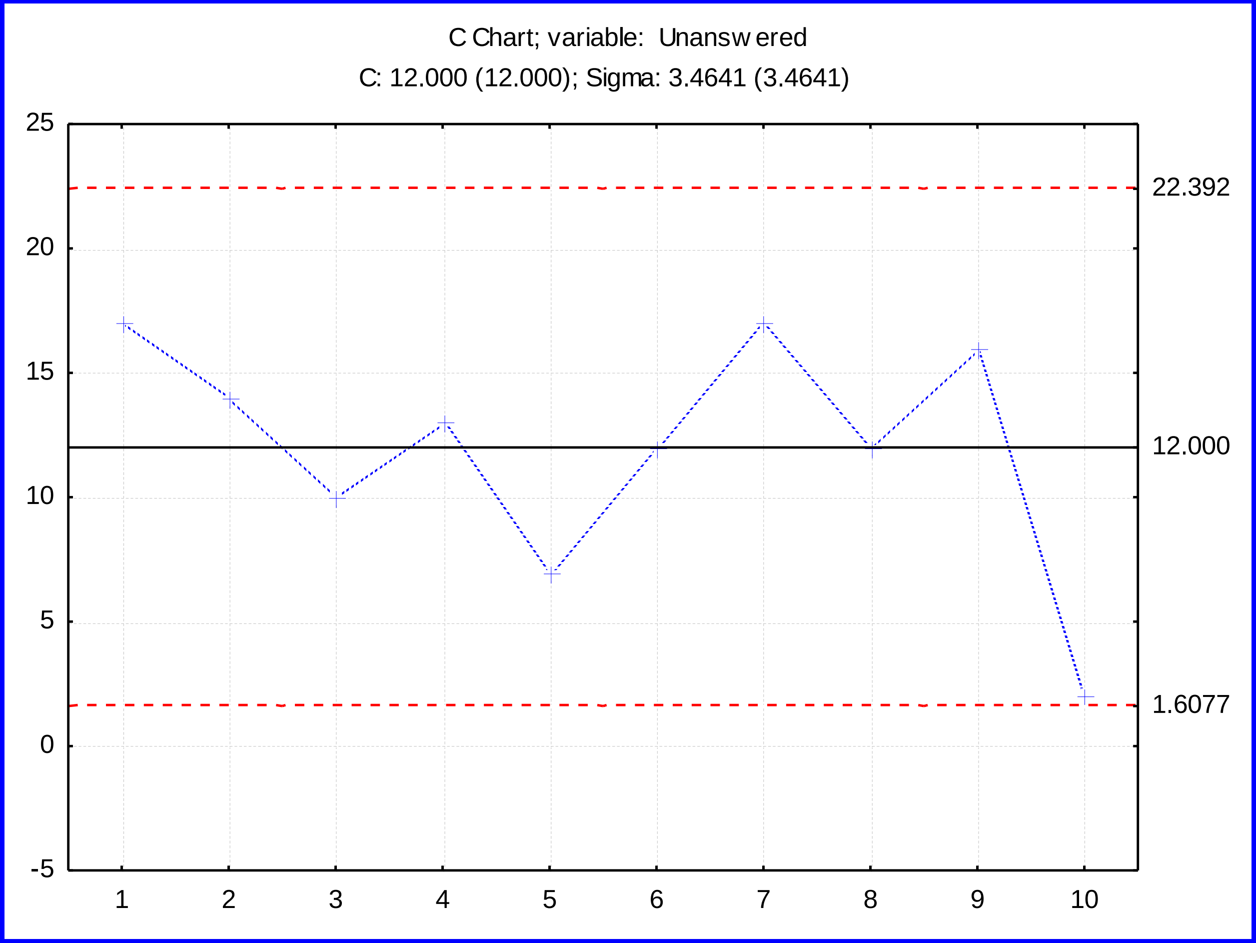
The average number of unanswered calls in a call center is 2 per hour (from earlier studies). Each week 6 hours are checked and considered as 1 sample. Prepare a *c* chart for checking stability of the process!

callcenter3.sta

week	# unanswered
1	17
2	14
3	10
4	13
5	7
6	12
7	17
8	12
9	16
10	2

Phase I or Phase II?

Statistics>Industrial Statistics &Six Sigma>
Quality Control Charts>Attributes > C...



C Chart; variable: Unansw ered
C: 12.000 (12.000); Sigma: 3.4641 (3.4641)
25
20
15
10
5
0
-5
22.392
12.000
1.6077
1
2
3
4
5
6
7
8
9
10

Control charts for occurrence of defects: *u* chart

The size of the sample may not be constant

E.g.

the car doors may not be of the same type,

the length of checked welding changes,

the number of pieces on days are different

the complexity of bills may be different,

the number of calls on different days is different

$$u_i = \frac{c_i}{n_i}$$

c_i # of defects for the sample *i*,

n_i size of sample *i* (m^2, #, m)

$$CL_u = \bar{u}$$

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$

$$LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$$

$$\bar{u} = \frac{\sum_i c_i}{\sum_i n_i}$$

n_i changes from sample to sample!

Example 27

The unit sample in Phase I contained 5 doors, 1.1 m^2 each.
20 unit samples were used to estimate the λ parameter of the process. The average number of defects for a sample (5·1.1 m^2) were found as 7.2.
Compute the parameters of the *u* chart!

$$CL_u = \bar{u} = \frac{7.2}{5 \cdot 1.1} = 1.309$$

$$UCL_u = 1.309 + 3\sqrt{\frac{1.309}{n}}$$

$$LCL_u = 1.309 - 3\sqrt{\frac{1.309}{n}}$$

n is the area of the door checked

if the area is 0.9 m2?

Example 20

The unit sample in Phase I contained 5 bills, with 15 candidate DPU each.
20 unit samples were used to estimate the λ parameter of the process. The average number of defects for a unit sample $(5 \cdot 15)$ were found as 7.2.

Compute the parameters of a sample in the *u* chart for 10 bills of complexity 10 candidate DPU!

$$\bar{u} = \frac{\sum_i c_i}{\sum_i n_i}$$

$$CL_u = \bar{u} = \frac{7.2}{5 \cdot 15} = 0.096$$

$$UCL_u = 0.096 + 3\sqrt{\frac{0.096}{n}}$$

$$LCL_u = 0.096 - 3\sqrt{\frac{0.096}{n}}$$

$$n = 10 \cdot 10 = 100$$

Demerit systems

The severity of different types of non-conformities may be different

- A: typing error in the payable amount (100)
- B: wrong deadline (50)
- C: typing error in the address of client (10)
- D: missing letter in the first name of the client (1)

$$D = 100c_A + 50c_B + 10c_C + c_D \qquad u = \frac{D}{n}$$

$$\bar{u} = 100\bar{u}_A + 50\bar{u}_B + 10\bar{u}_C + \bar{u}_D$$

35. példa

In Phase I mistype errors of 5 invoices were studied, all contained 50 characters each, this was a sample.
20 samples were used to estimate the λ parameter of the process. The average number of mistype was found as 7.2 for a sample (5·50 characters).

$$CL_u = \bar{u} = \frac{7.2}{5 \cdot 50} = 0.0288$$

$$UCL_u = 0.0288 + 3\sqrt{\frac{0.0288}{n}}$$

$$LCL_u = 0.0288 - 3\sqrt{\frac{0.0288}{n}}$$

n is the number of characters in an invoice
E.g. for 10 invoices, containing 30 characyetrs each?

Comparison of variables and attributes control charts

variables: continuous random variable

attributes: discrete random variable

The variables charts:

- offer more information, more sensitive to changes, the signal the special causes (e.g. shift) before defectives are manufactured, since the specification limits are not necessarily reached when control limits are exceeded.
- require much smaller sample size, but the measurement is usually more expensive then deciding on attributes, and the former is not always applicable.

variables data

data collected in groups:	X-bar/R
individual data:	I/MR, X/MR

attribute data

nonconforming items

sample size is constant:	np or p
sample size is changing:	p

defects

sample size is constant:	c
sample size is changing:	u