CONTROL CHARTS

- variables
- attributes

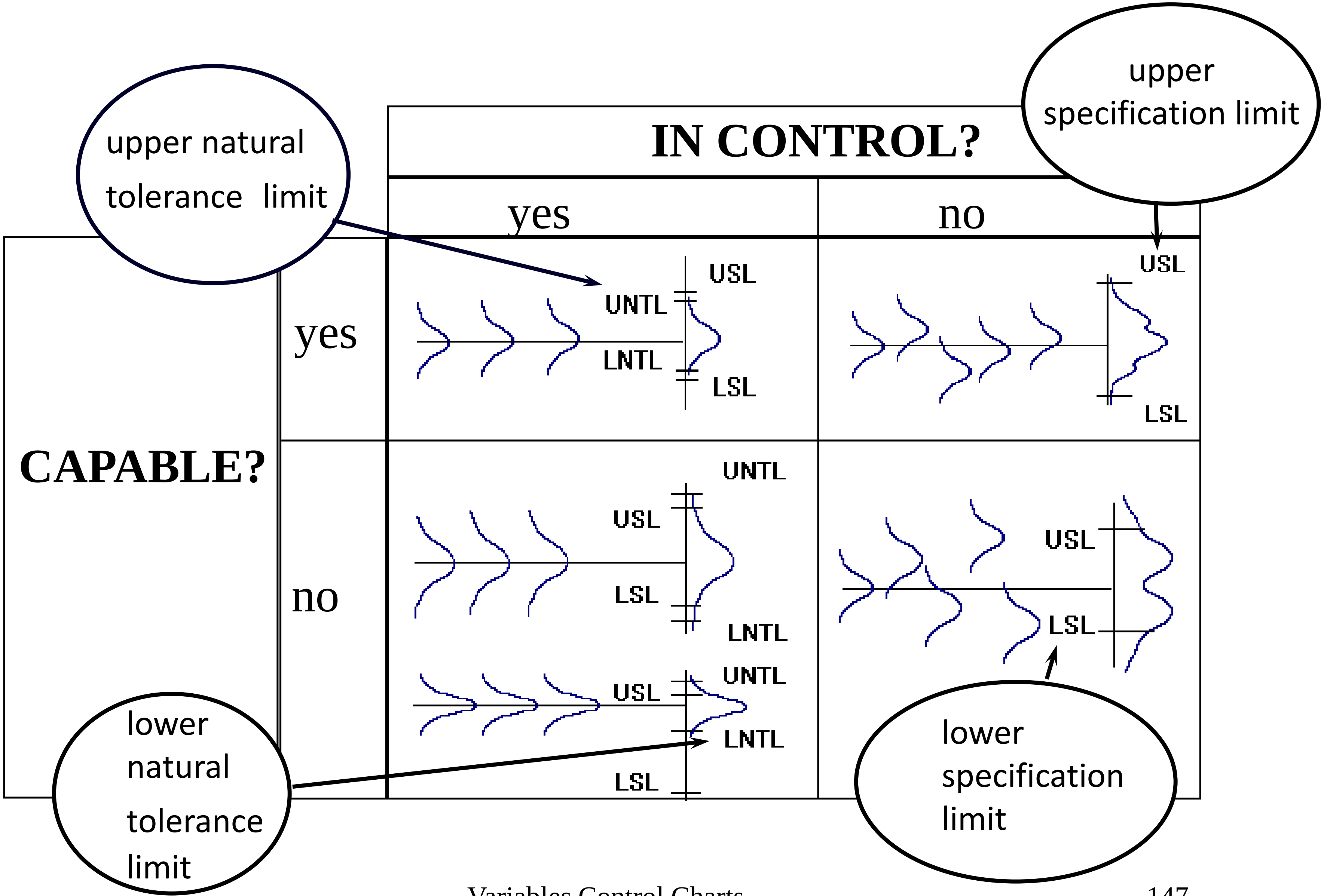
common (chance) cause

left to chance

specific (assignable) cause

identified and eliminated

Role of Quality Engineering



The process is considered in control if the variation is stable in time, there are no assignable causes.

If the process is in statistical control its future behavior may be predicted based on past data. This means we may give the probability of finding the process variable within certain limits (Shewhart, 1931).

Variables control charts

Example 17

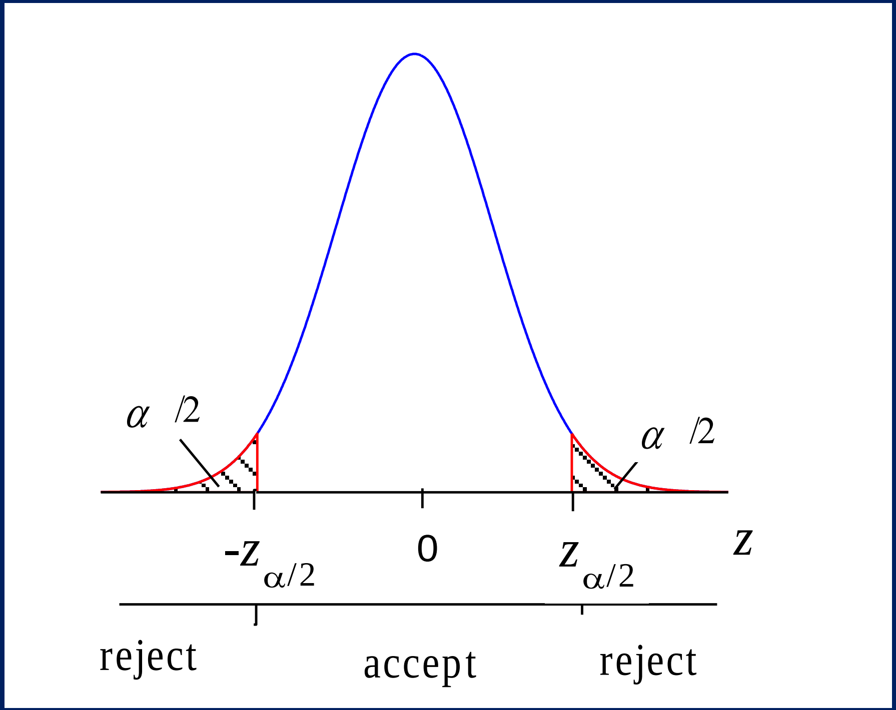
The assumed expected value of the mass of packages produced by an automatic machine is **250 g**, the known variance of the process is **1 g^2**.
The mean of the sample of element 5 taken from the process is:

$$\bar{x} = 249.6 \text{ g}$$

May the assumption on the expected value of the mass of packages be accepted if the allowed probability for the error of first kind is α=0.05?

The null hypothesis: $H_0: \; E(x) = \mu_0 = 250$

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$



The region of acceptance:

$$P\left(-z_{\alpha/2} < z_0 \le z_{\alpha/2} \middle| H_0\right) = 1 - \alpha$$

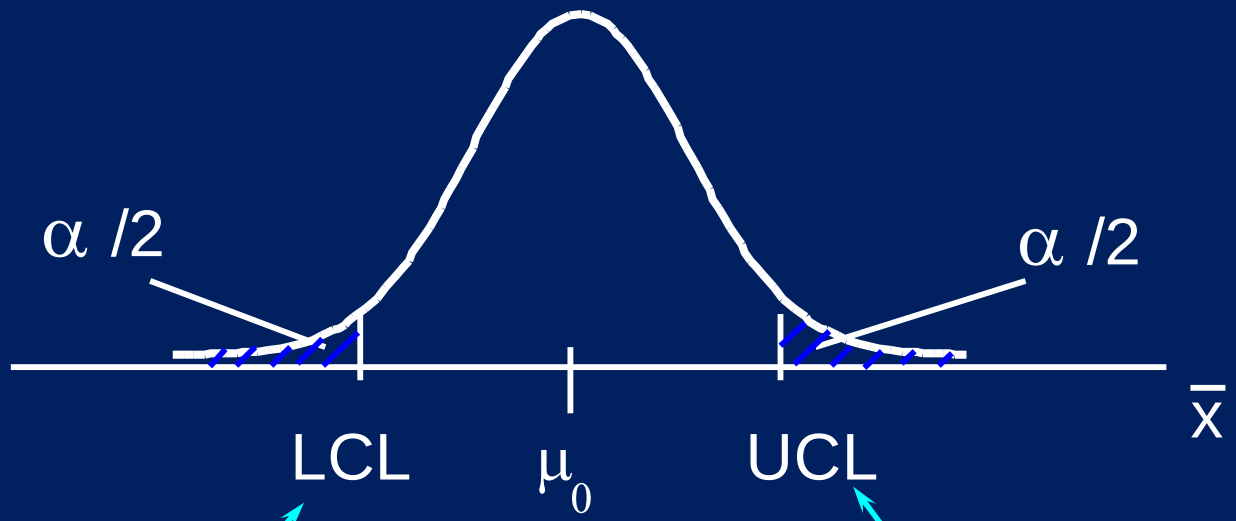
$$\mu_0 - z_{\alpha/2}\,\sigma/\sqrt{n} < \bar{x} < \mu_0 + z_{\alpha/2}\,\sigma/\sqrt{n}$$

$$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} < \mu_0 < \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$$

the confidence interval contains the hypothetisized μ_0

$H_0: \quad E(x) = \mu_0 = 250$

$$\mu_0 - z_{\alpha/2}\,\sigma/\sqrt{n} < \bar{x} < \mu_0 + z_{\alpha/2}\,\sigma/\sqrt{n}$$



LCL: lower control limit:

$\mu_0 - z_{\alpha/2}\sigma/\sqrt{n}$

UCL: upper control limit

$\mu_0 + z_{\alpha/2}\sigma/\sqrt{n}$

$$H_0: \quad E(x) = \mu_0 = 250$$

The region of acceptance:

$$\mu_0 - z_{\alpha/2}\,\sigma\big/\sqrt{n} < \bar{x} < \mu_0 + z_{\alpha/2}\,\sigma\big/\sqrt{n}$$

$$UCL = \bar{x}_{\text{upper}} = \mu_0 + z_{\alpha/2}\sigma/\sqrt{n} =$$

$$z_{\alpha/2} =$$

$$LCL = \bar{x}_{\text{lower}} = \mu_0 - z_{\alpha/2}\sigma/\sqrt{n} =$$

Decision:

The region of acceptance:

$$\mu_0 - z_{\alpha/2}\,\sigma\big/\sqrt{n} < \bar{x} < \mu_0 + z_{\alpha/2}\,\sigma\big/\sqrt{n}$$

Take samples (subgroup) time to time and plot their mean as a function of time!

→ control chart

- in statistical control: continue
- out of control: stop the process

The intervention is usually expensive (the manufacturing line is stopped), thus the chance for false alarm is to be diminished:

$z_{\alpha/2}$ =3 (the so called $\pm 3\sigma$ limit),

then α=0.0027, that is the chance for erroneous decision is about three from among one thousand.

$$\mu_0 - 3\sigma/\sqrt{n} < \bar{x} < \mu_0 + 3\sigma/\sqrt{n}$$

LCL ***UCL***

The region of acceptance:

$$\mu_0 - 3\sigma/\sqrt{n} < \bar{x} < \mu_0 + 3\sigma/\sqrt{n}$$

Problem 1

μ_0 and σ are not known (we do not know the reference to which the process is to be compared)

→ estimation from a large sample

Problem 2

We may not be sure if the process used for estimating μ and σ is in control

→ check using control chart

Phase I: establishing stability and control limits

Phase II: on-going control using the previously established control limits

The X-bar - Range chart

n (typically *n*=3 - 5) samples are taken from the process time to time. The mean and the range of the sample is computed:

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j \qquad R = \left| x_{\max} - x_{\min} \right|$$

An R_i range and $\bar{x}_i$ mean is found for the sample *i*.

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \quad \text{where} \quad \bar{R} = \frac{1}{m}\sum_i R_i$$

Construction of the X-bar chart

Phase I

$$CL_{\bar{x}} = \bar{\bar{x}} = \frac{1}{m}\sum_i \bar{x}_i$$

(m is the number of samples, $\bar{x}_i$ is the mean of the i-th sample)

$$UCL_{\bar{x}} = \bar{\bar{x}} + \frac{3\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}$$

(upper control limit)

$$LCL_{\bar{x}} = \bar{\bar{x}} - \frac{3\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R}$$

(lower control limit)

Phase II (on-going control)

$\bar{\bar{x}}$ and $\bar{R}$

from Phase I, that is the center line and control limits are given

Construction of the range (R) chart

$$\mathrm{H}_0 : Var(x) = \sigma_0^2$$

Phase I

$$CL_R = \overline{R} = \frac{1}{m}\sum_i R_i \qquad \hat{\sigma}_R = d_3\hat{\sigma} = \frac{d_3\overline{R}}{d_2} = \frac{(D_4 - 1)\overline{R}}{3}$$

The control limits for the $\pm 3\sigma$ rule:

$$UCL_R = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3\frac{d_3\overline{R}}{d_2} = \overline{R}\left(1 + 3\frac{d_3}{d_2}\right) = D_4\overline{R}$$

$$LCL_R = \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3\frac{d_3\overline{R}}{d_2} = \overline{R}\left(1 - 3\frac{d_3}{d_2}\right) = D_3\overline{R}$$

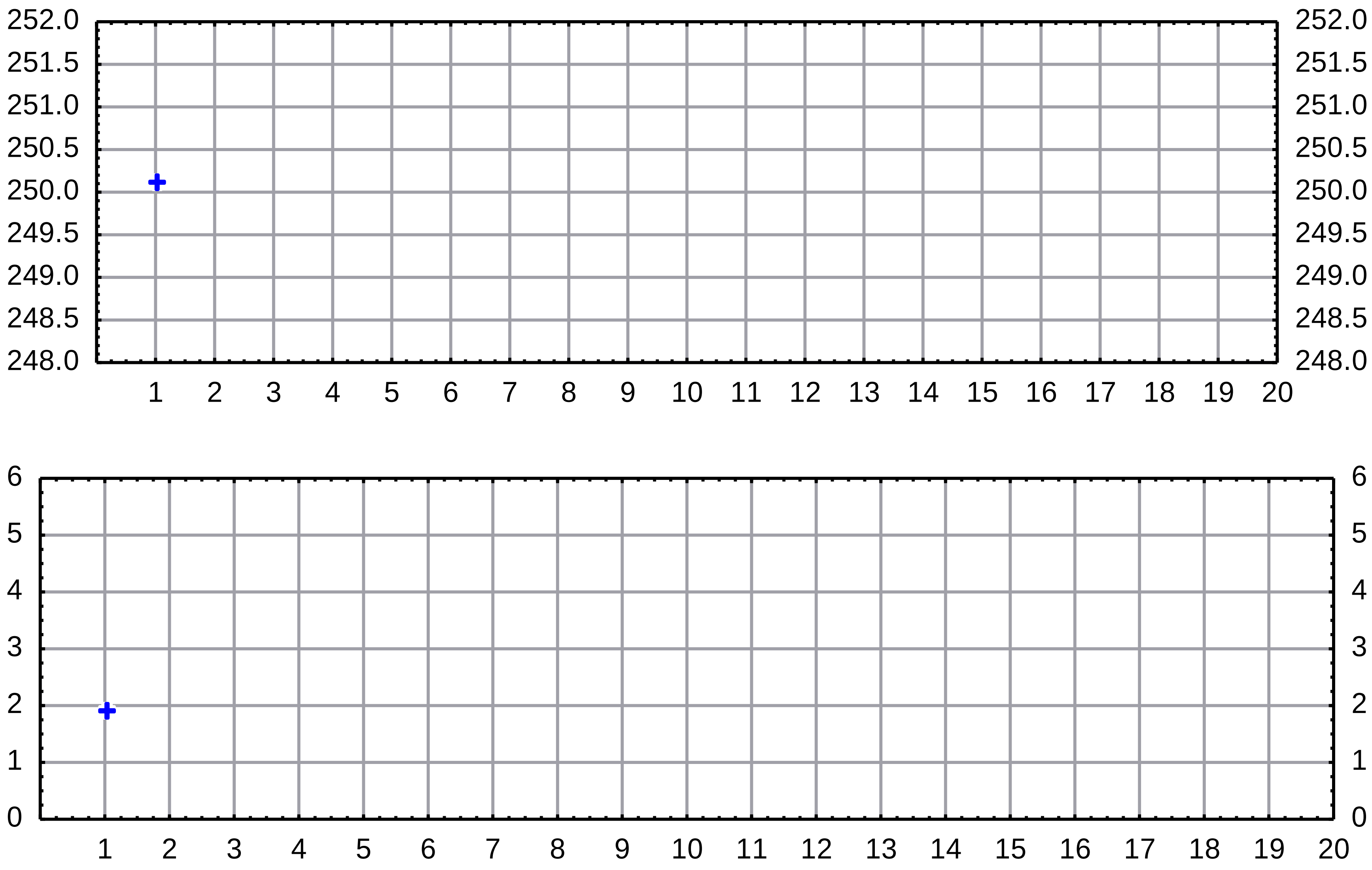
If negative value is obtained for *LCL*, it is to be set as zero

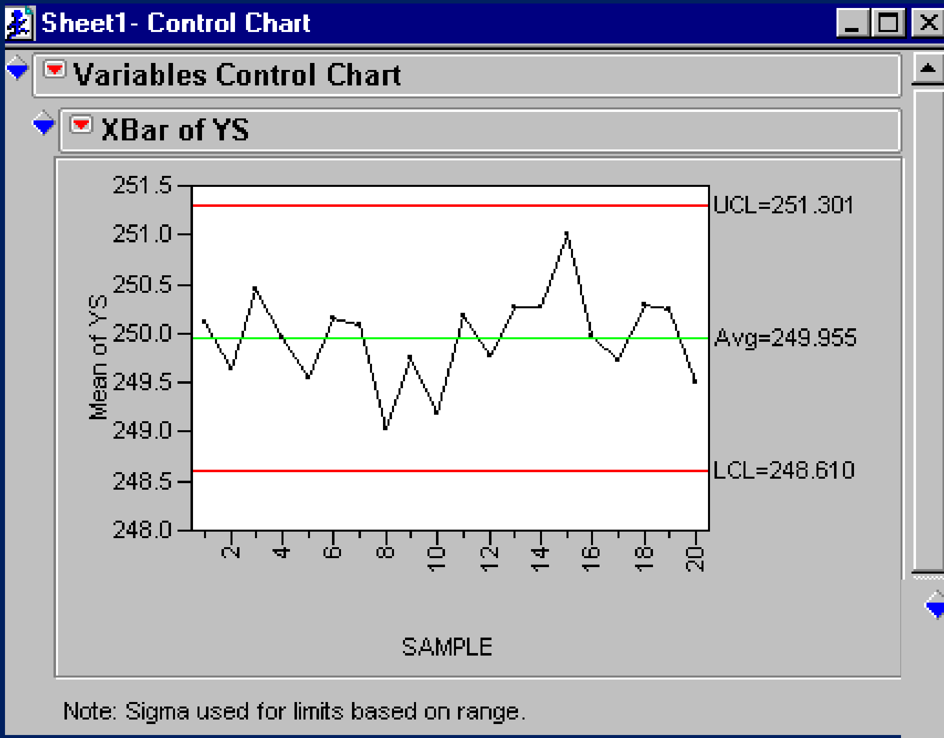
n	d_2	d_3	c_4	A_2	A_3	B_3	B_4	D_3	D_4
2	1.128	0.853	0.7979	1.880	2.659	0	3.267	0	3.267
3	1.693	0.886	0.8862	1.023	1.954	0	2.568	0	2.574
4	2.059	0.880	0.9213	0.729	1.628	0	2.266	0	2.282
5	2.326	0.864	0.9400	0.577	1.427	0	2.089	0	2.114

Example 18

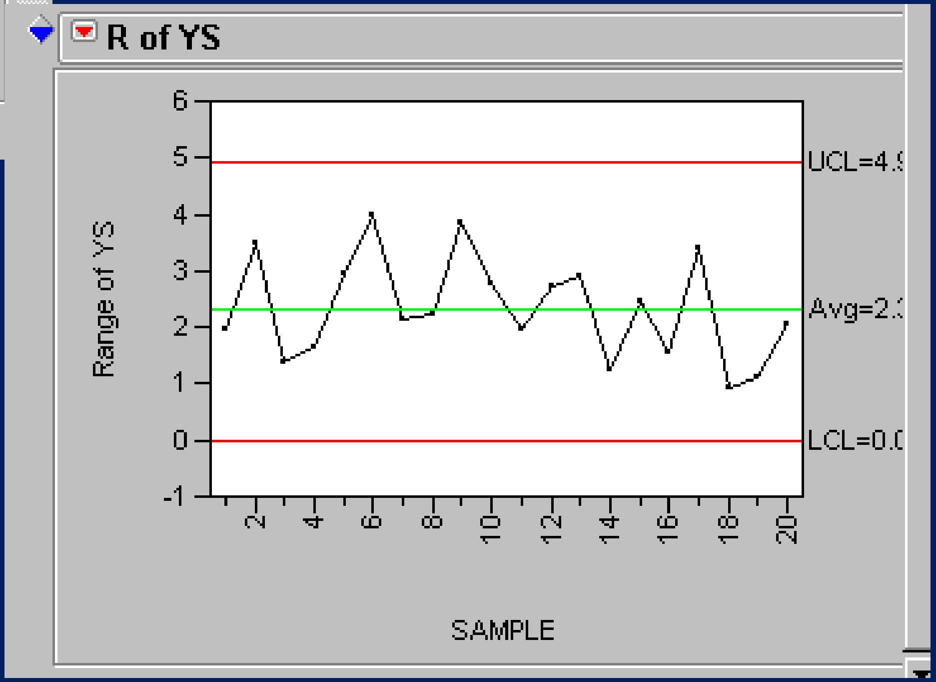
Prepare an X-bar/R chart using the data in the table!

i	measured sample elements					mean	median	*R*
1	251.25	249.67	250.15	250.22	249.30	250.118	250.150	1.950
2	247.56	249.84	251.04	249.47	250.25			
3	251.47	250.23	250.07	250.12	250.37			
4	249.35	249.77	249.29	250.92	250.44	249.954	249.770	1.630
5	249.09	251.09	248.14	248.51	250.90	249.546	249.090	2.950
6	251.59	248.13	250.06	248.92	252.09	250.158	250.060	3.960
7	250.61	249.55	249.23	249.61	251.39	250.078	249.610	2.160
8	249.95	247.74	249.40	248.88	249.16	249.026	249.160	2.210
9	247.74	249.42	249.59	251.59	250.36	249.740	249.590	3.850
10	247.89	250.65	249.61	249.08	248.72	249.190	249.080	2.760
11	249.26	250.08	251.22	250.08	250.26	250.180	250.080	1.960
12	249.83	249.46	248.83	251.56	249.16	249.768	249.460	2.730
13	250.36	250.10	251.68	250.36	248.78	250.256	250.360	2.900
14	250.71	250.26	250.18	249.47	250.72	250.268	250.260	1.250
15	250.50	252.36	251.52	249.91	250.75	251.008	250.750	2.450
16	250.11	250.87	249.31	249.93	249.63	249.970	249.930	1.560
17	248.81	249.65	248.08	250.57	251.48	249.718	249.650	3.400
18	249.90	249.81	250.59	250.38	250.74	250.284	250.380	0.930
19	250.88	249.79	249.85	250.11	250.61	250.248	250.110	1.090
20	249.27	248.61	250.64	249.43	249.60	249.510	249.430	2.030
mean						249.955	249.850	2.333


252.0
251.5
251.0
250.5
250.0
249.5
249.0
248.5
248.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20



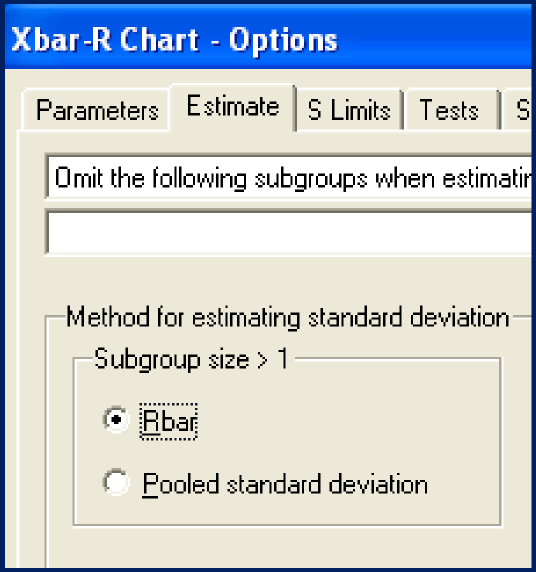
Open Data Table: Chartdata2.xls
Graph>ControlChart
Process: YS
Sample Label: Sample



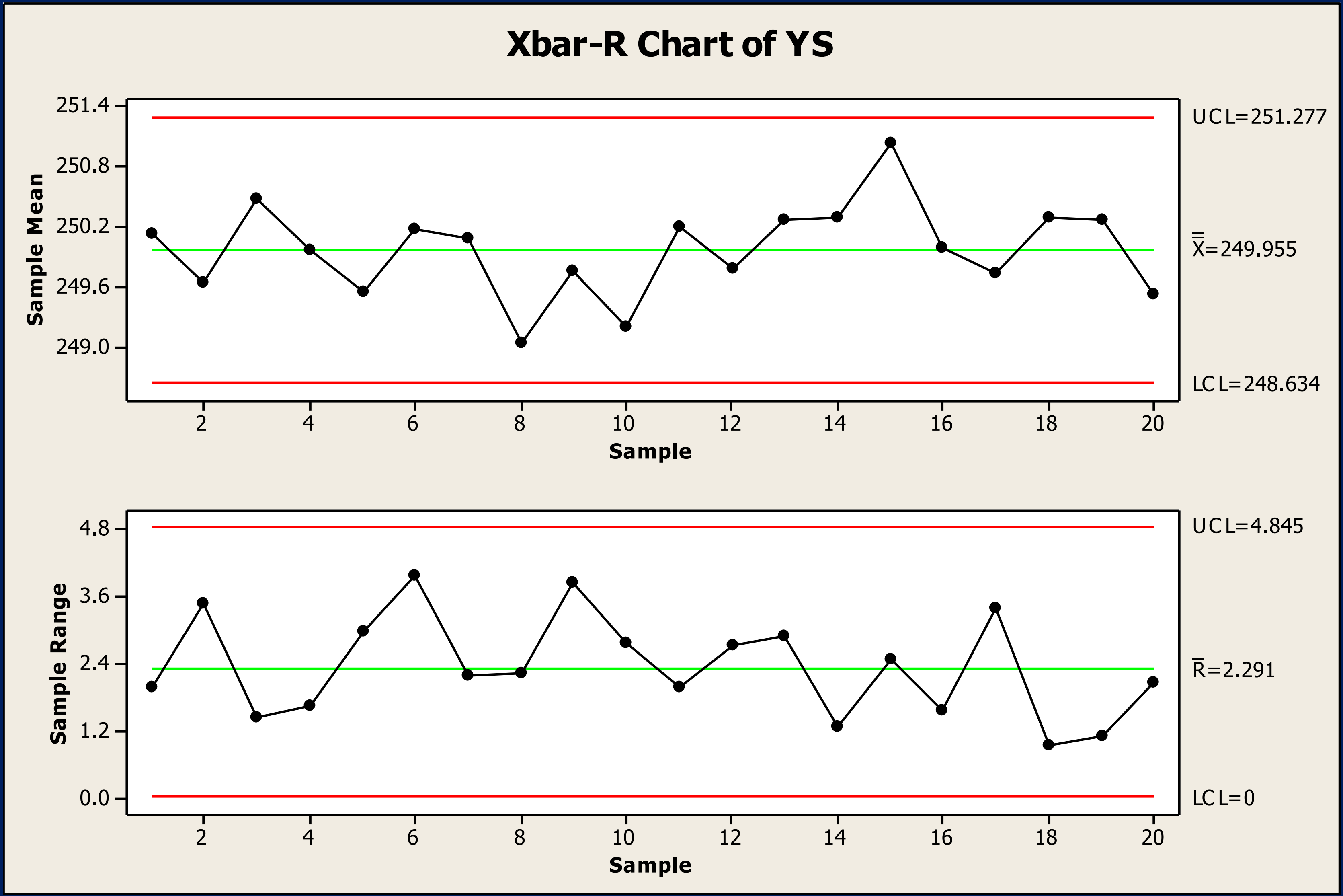
Example 18

Prepare an X-bar/R chart using the YS column of the cpdata1.mtw data file!

Phase I or Phase II?



Minitab>Stat>Control Charts>
Variables Charts for Subgroups>Xbar-R

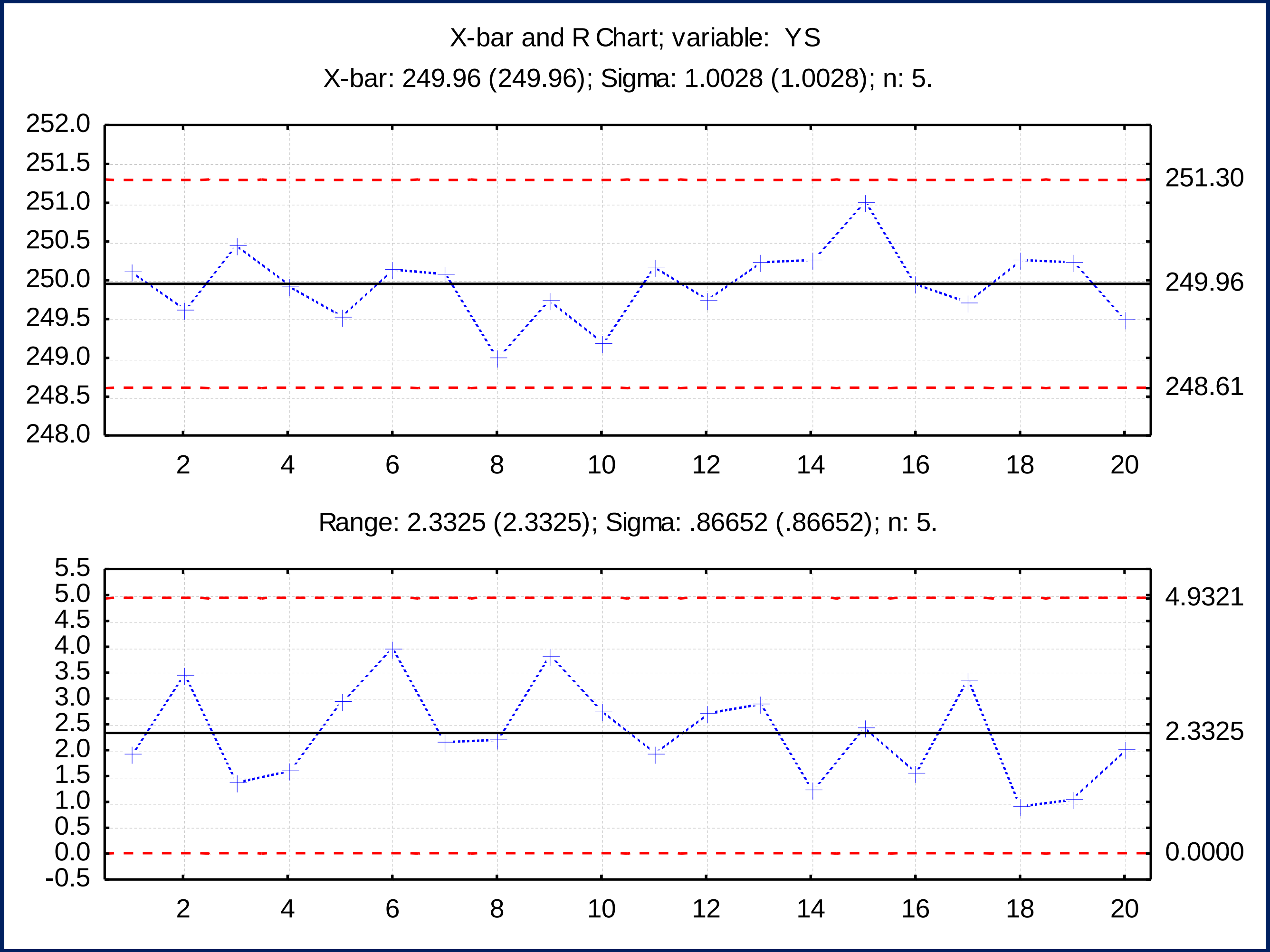

Xbar-R Chart of YS
Sample Mean
251.4
250.8
250.2
249.6
249.0
UCL=251.277
X̿=249.955
LCL=248.634
2
4
6
8
10
12
14
16
18
20
Sample
Sample Range
4.8
3.6
2.4
1.2
0.0
UCL=4.845
R̄=2.291
LCL=0
2
4
6
8
10
12
14
16
18
20
Sample

Example 11

Prepare an X-bar/R chart using the YS column of the cpdata1.sta data file!

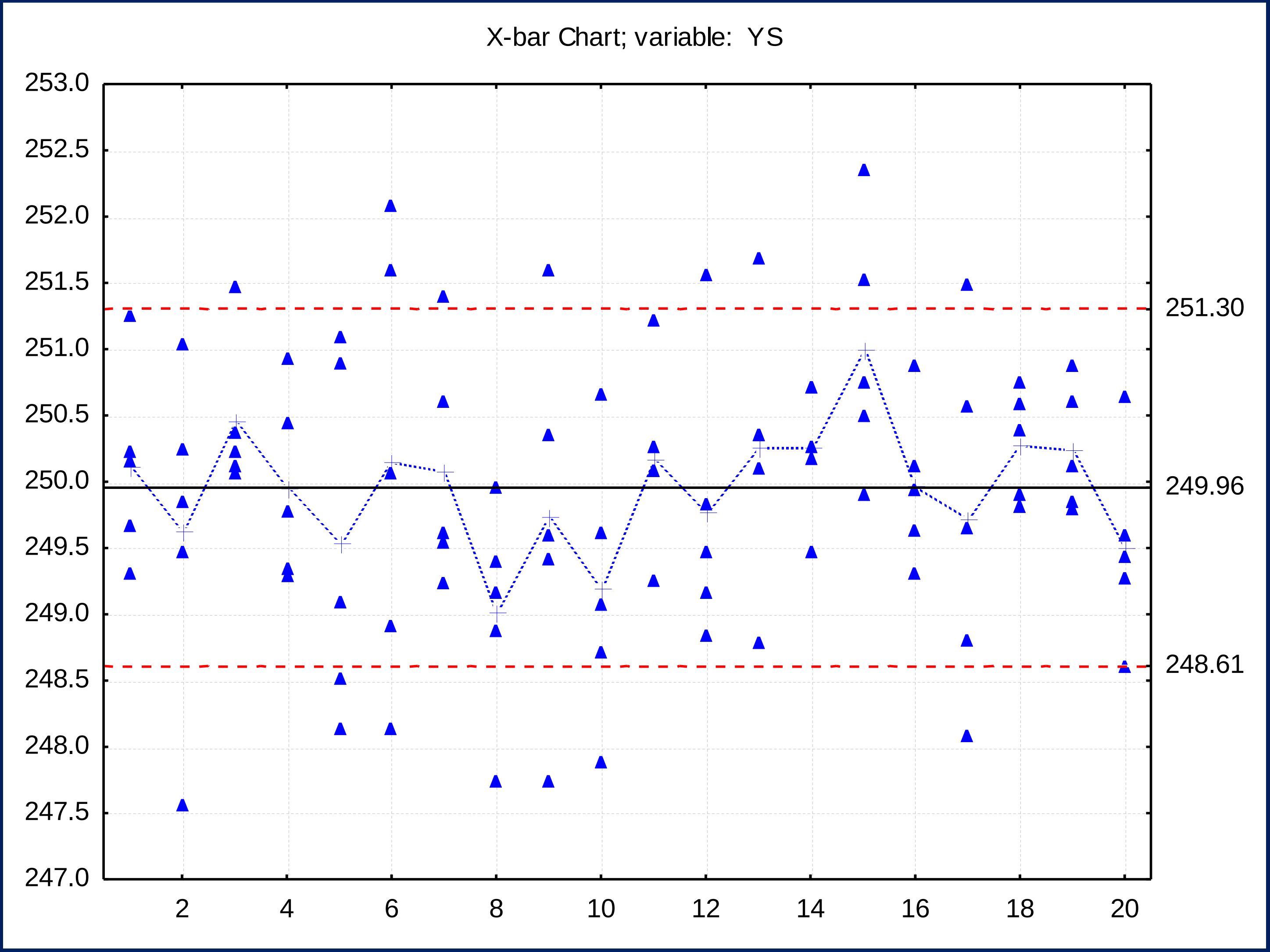
Phase I or Phase II?

Open cpdata1.sta
Statistics>Industrial Statistics>Quality Control Charts
X-bar & R chart for variables
Variables: YS, Sample

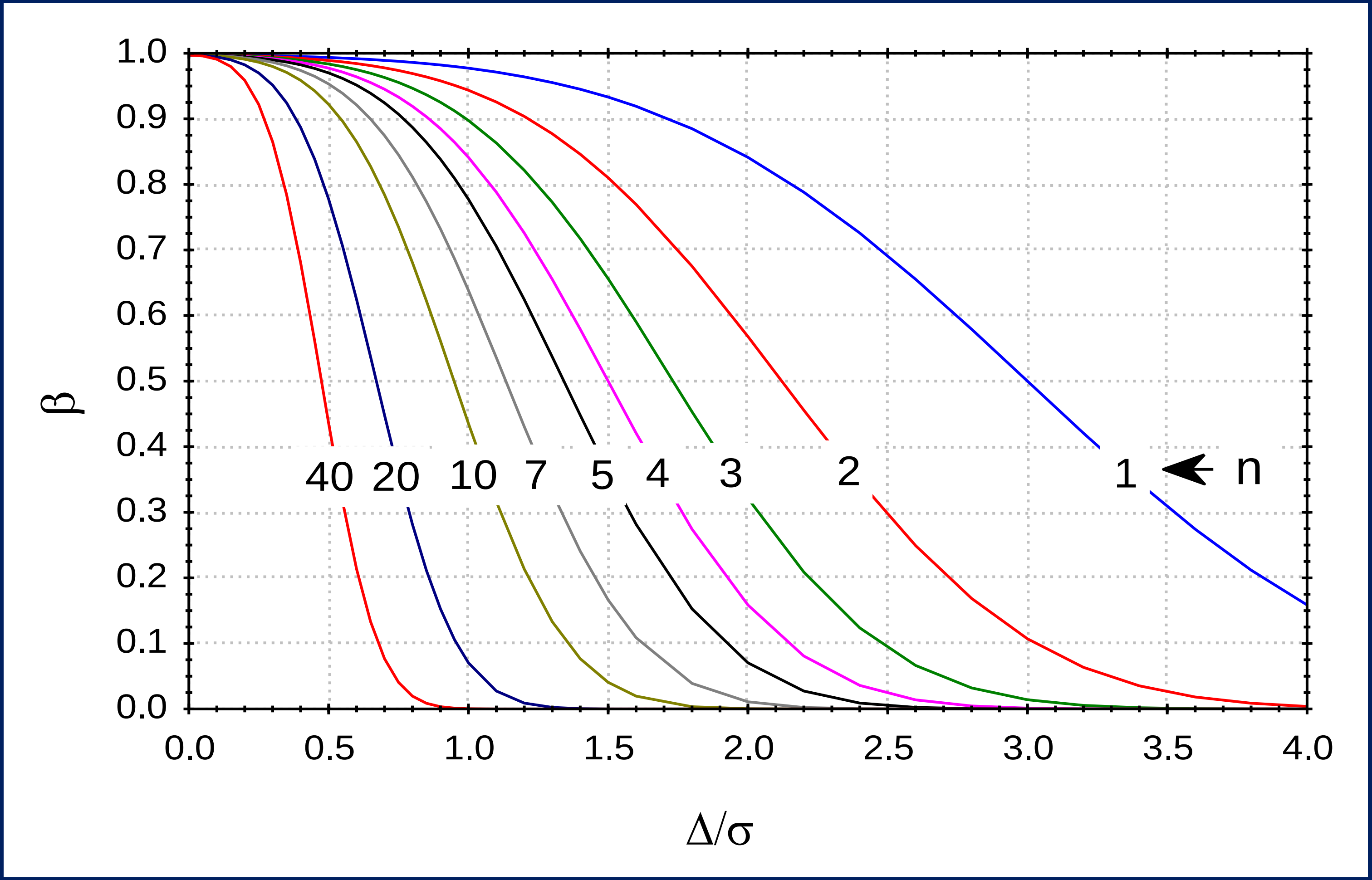


X-bar and R Chart; variable: YS
X-bar: 249.96 (249.96); Sigma: 1.0028 (1.0028); n: 5.
252.0
251.5
251.0
250.5
250.0
249.5
249.0
248.5
248.0
251.30
249.96
248.61
2
4
6
8
10
12
14
16
18
20
Range: 2.3325 (2.3325); Sigma: .86652 (.86652); n: 5.
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
4.9321
2.3325
0.0000
2
4
6
8
10
12
14
16
18
20

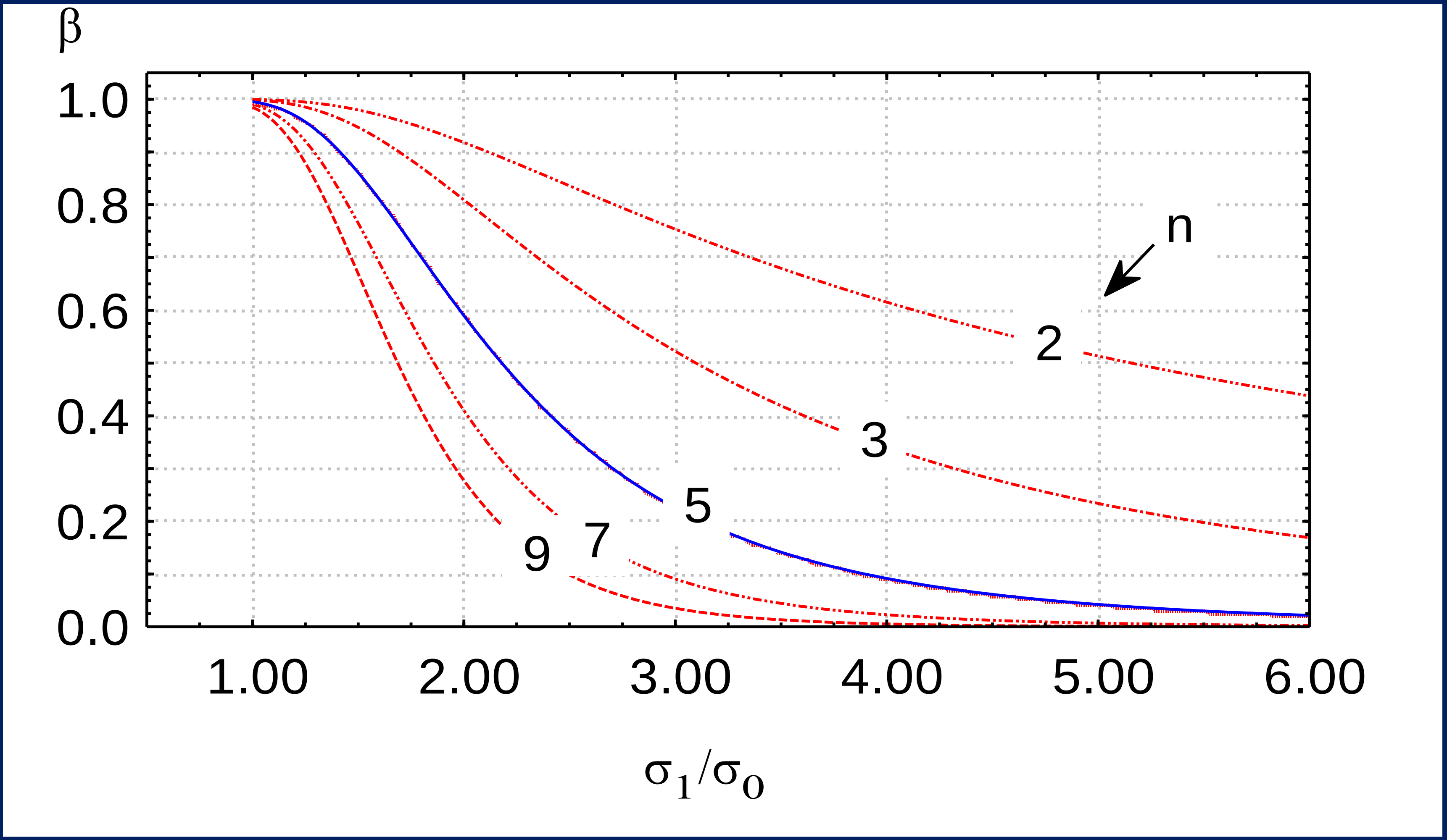
The control limits on the X-bar chart refer to the mean, not to single measurement values!



Operating Characteristic (OC) curve for the X-bar chart (α=0.0027)



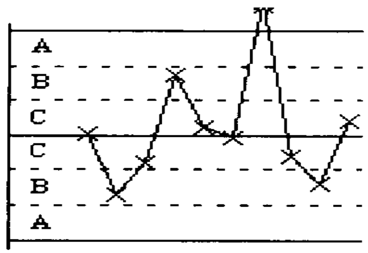
Operating Characteristic (OC) curve for the R chart (±3σ, that is α=0.0027)



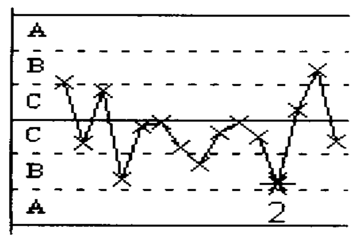
The Western Electric algorithmic rules (run tests)

Western Electric rules (runs test)

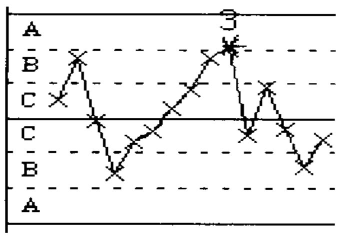
1. One point beyond Zone A



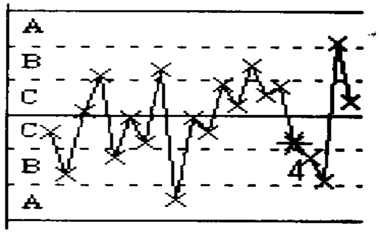
2. 9 points in Zone C or beyond (on one side of central line)



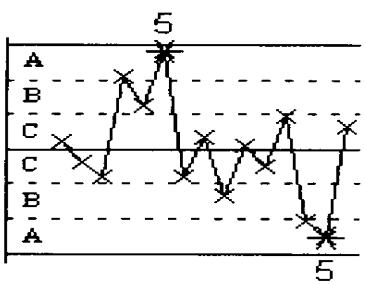
3. 6 points in a row steadily increasing or decreasing



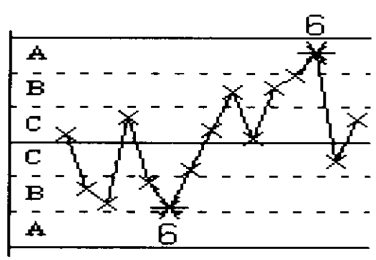
4. 14 points in a row alternating up and down



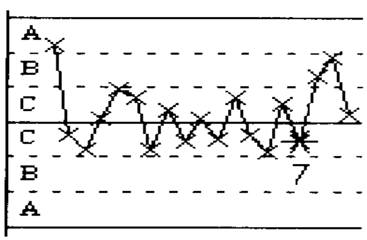
5. 2 out of 3 points in a row in Zone A or beyond



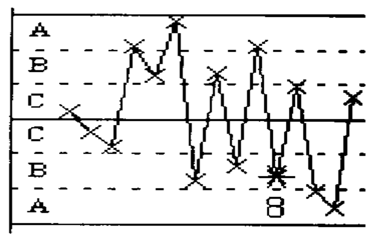
6. 4 out of 5 points in a row in Zone B or beyond



7. 15 points in a row in Zone C (above and below the center line)



8. 8 points in a row in Zone B, A, or beyond, on either side of the center line (without points in Zone C)



Example 18

Prepare an X-bar/R chart using the YS5 column of the cpdata1.mtw data file, making use of Western Electric rules!

The center of the process is 250., the variance is 1.0 as obtained in Phase I.

Phase I or Phase II?

Minitab>Stat>Control Charts>Variables Charts for Subgroups>Xbar-R

Xbar-R Options>Parameters

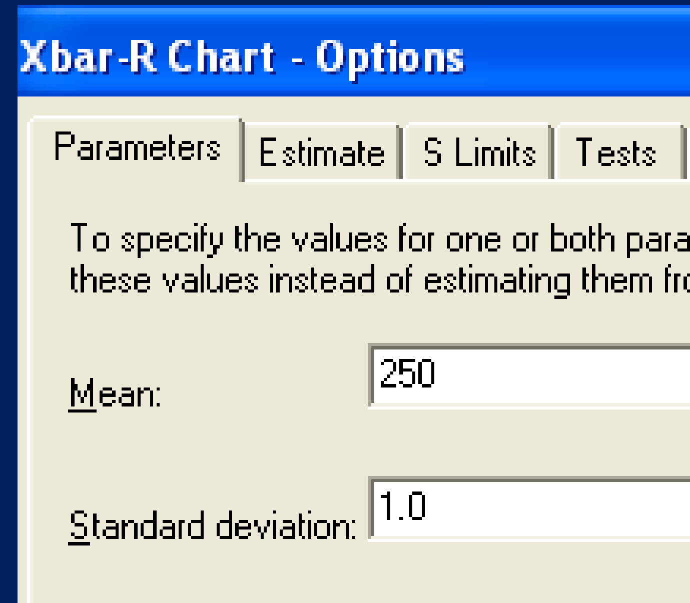
>Tests

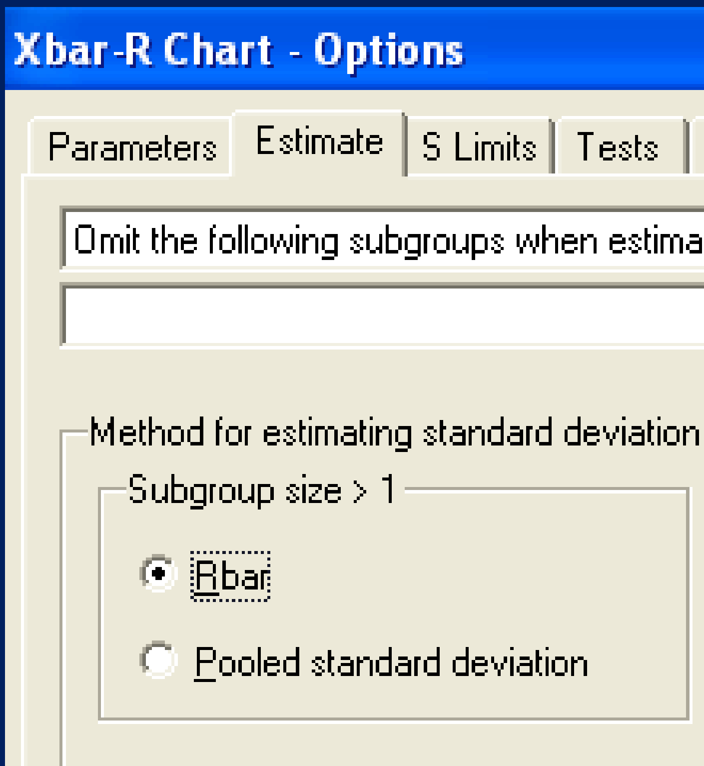
Minitab>Stat>Control Charts>Variables Charts for Subgroups>Xbar-R

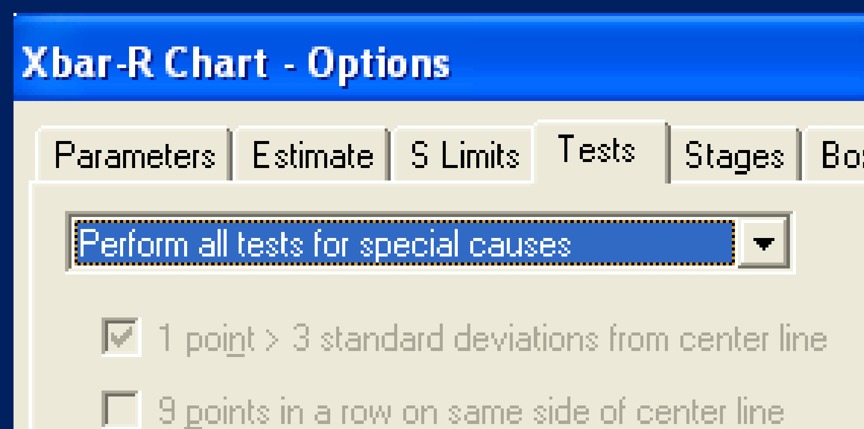
Xbar-R Options>Parameters

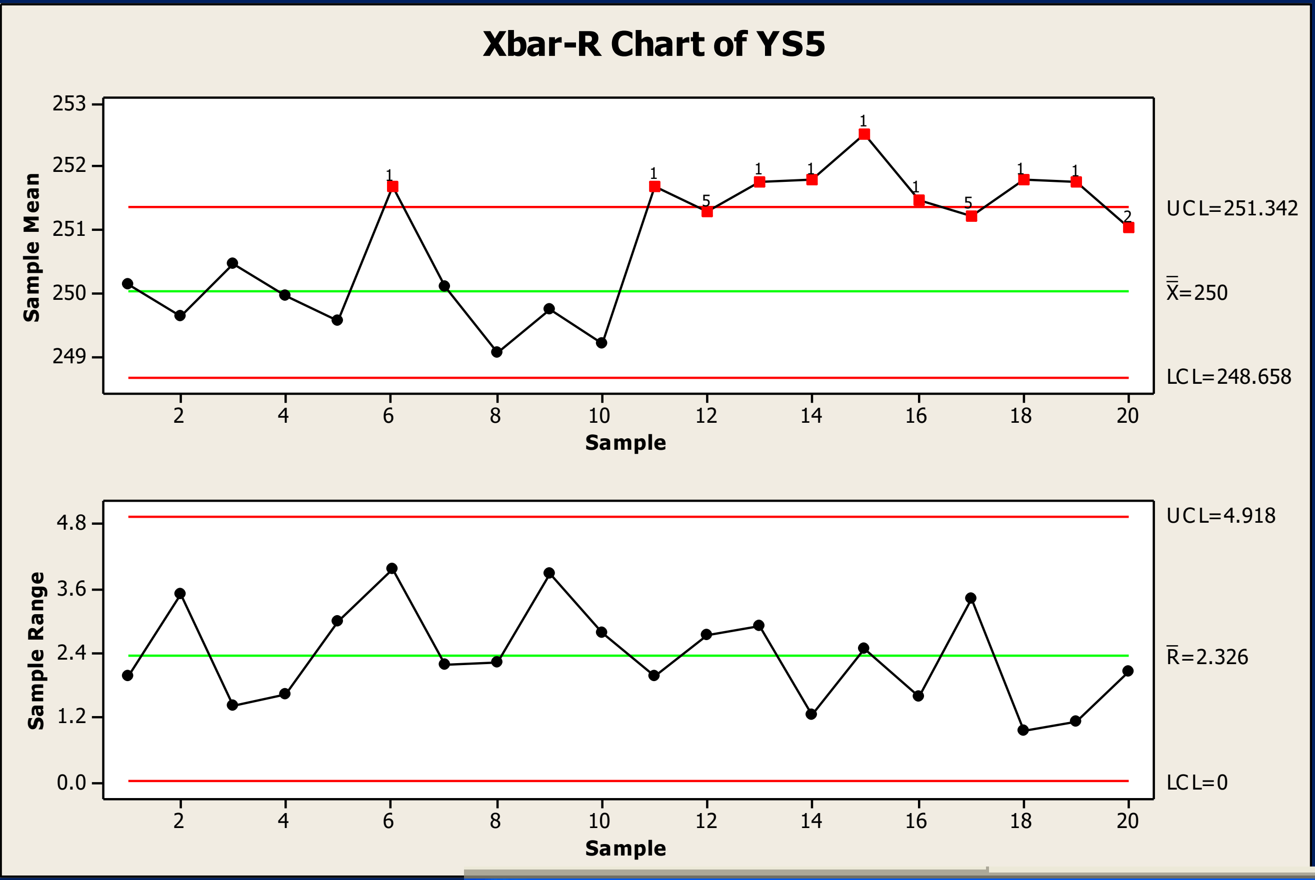
>Estimate: Rbar

>Tests









Session

Test Results for Xbar Chart of YS5

```
TEST 1. One point more than 3.00 standard deviations from
Test Failed at points:  6, 11, 13, 14, 15, 16, 18, 19

TEST 2. 9 points in a row on same side of center line.
```

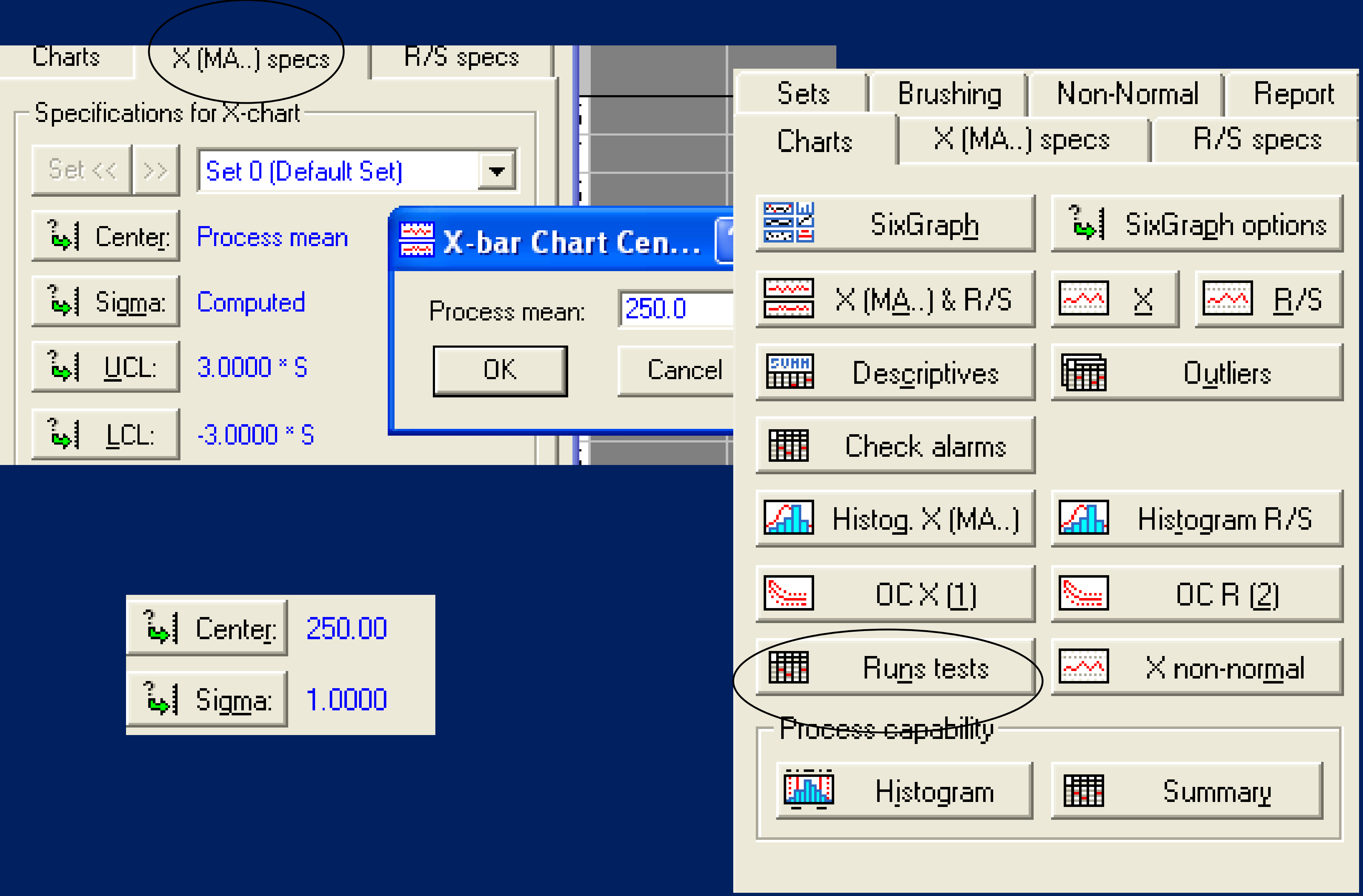
Example 12

Prepare an X-bar/R chart using the YS5 column of the cpdata1.sta data file, making use of Western Electric rules!

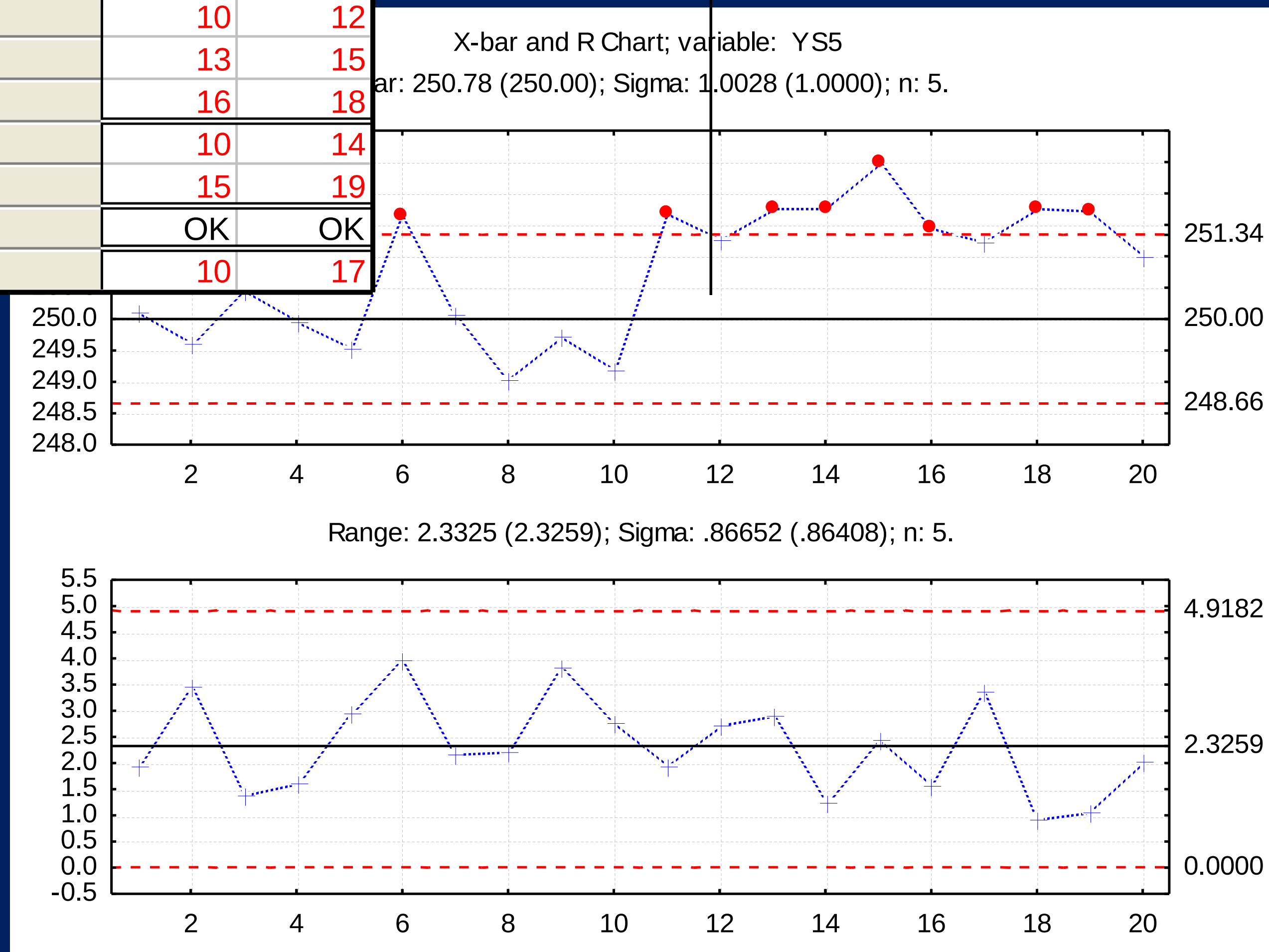
The center of the process is 250., the variance is 1.0 as obtained in Phase I.

Phase I or Phase II?

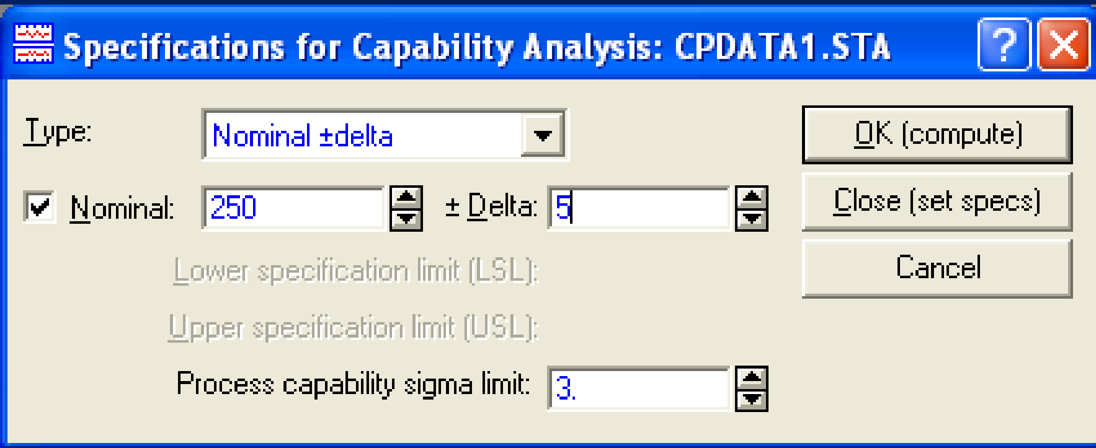
Statistics>Industrial Statistics>Quality Control Charts
X-bar & R chart for variables
Variables: YS, Sample
Runs test

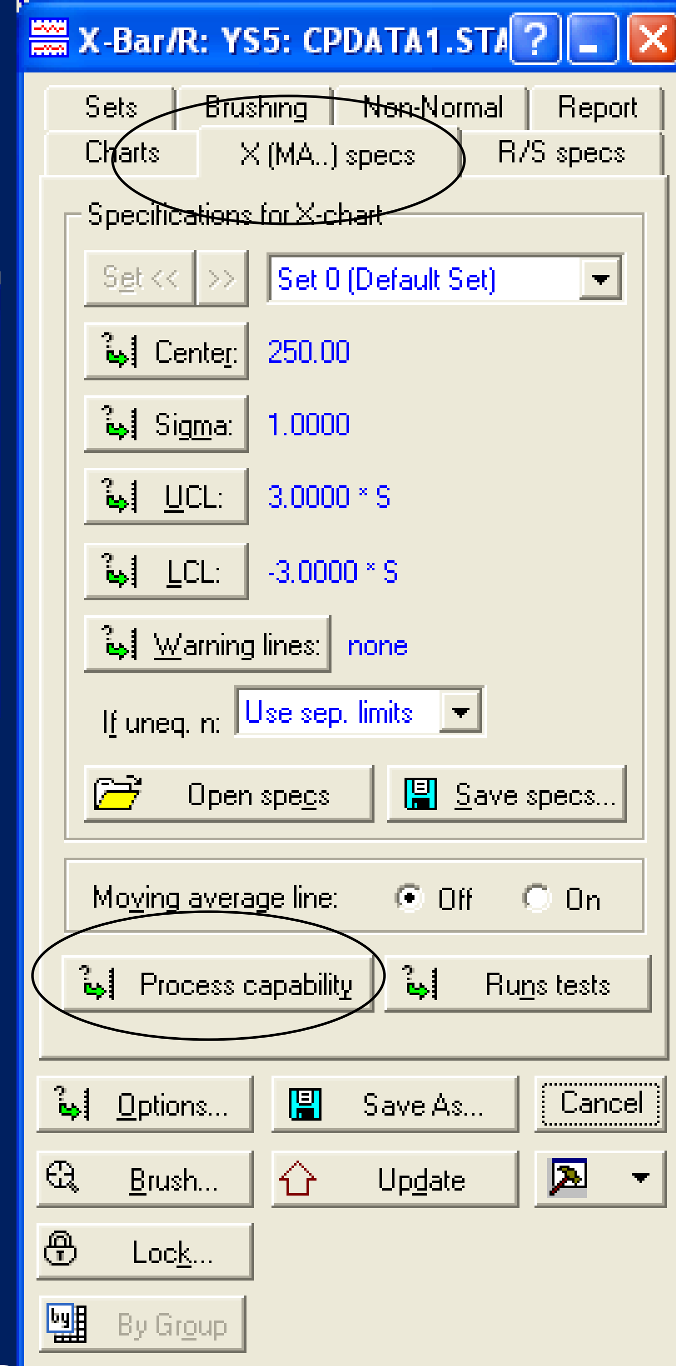

Charts
X (MA..) specs
R/S specs
Specifications for X-chart
Set <<
>>
Set 0 (Default Set)
Center:
Process mean
Sigma:
Computed
UCL:
3.0000 * S
LCL:
-3.0000 * S
X-bar Chart Cen...
Process mean:
250.0
OK
Cancel
Center:
250.00
Sigma:
1.0000
Sets
Brushing
Non-Normal
Report
Charts
X (MA..) specs
R/S specs
SixGraph
SixGraph options
X (MA..) & R/S
X
R/S
Descriptives
Outliers
Check alarms
Histog. X (MA..)
Histogram R/S
OC X (1)
OC R (2)
Runs tests
X non-normal
Process capability
Histogram
Summary

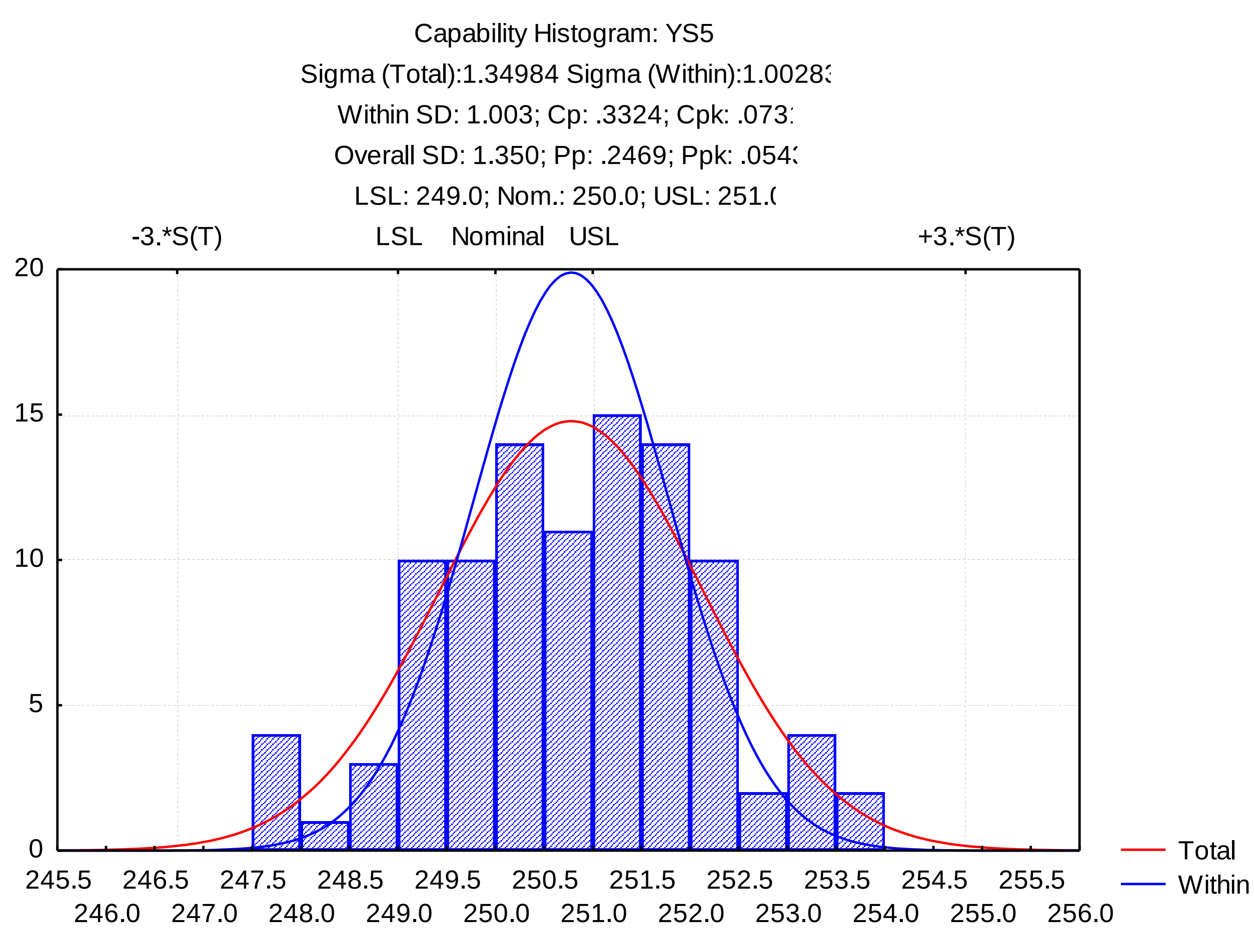
Zones A/B/C: 3.000/2.000/1.000 * Sigma Tests for special causes (runs rules)	YS5 ; Runs Tests (CPDATA1.STA) X-bar Chart Center line: 250.000000 Sigma: 0.448477 from sample	to sample
9 samples on same side of center	11	19
6 samples in row in/decreasing	OK	OK
14 samples alternating up & down	OK	OK
2 of 3 samples in Zone A or beyond	10	12
	13	15
	16	18
4 of 5 samples in Zone B or beyond	10	14
	15	19
15 samples in Zone C	OK	OK
8 samples beyond Zone C	10	17



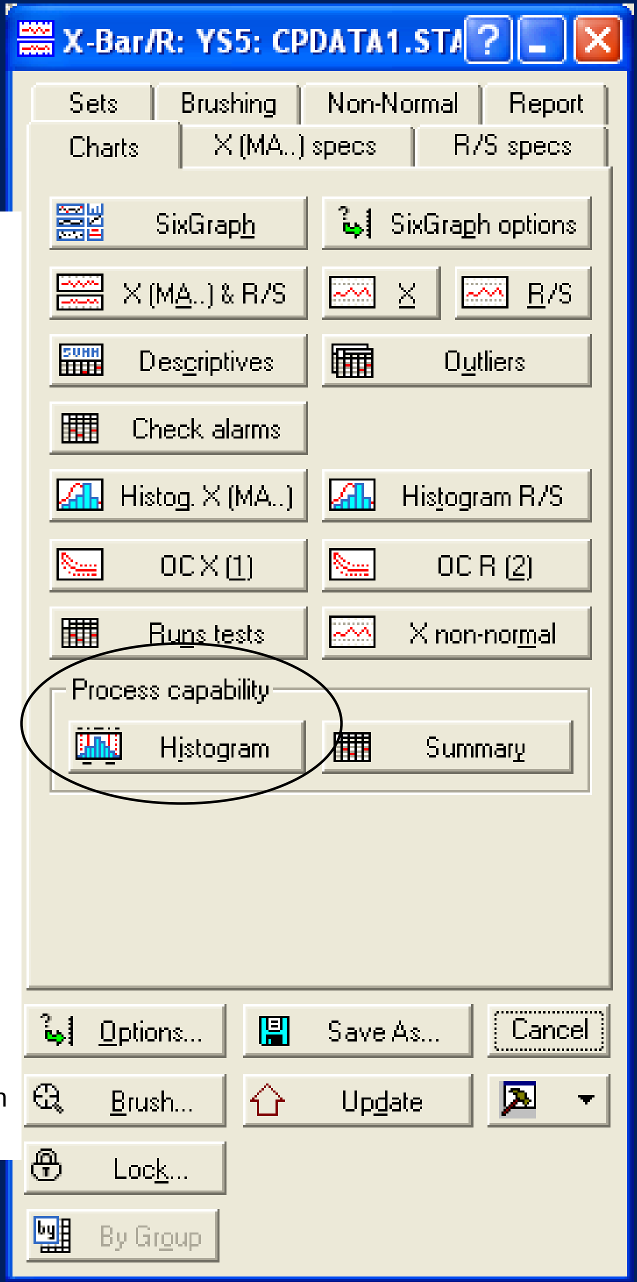
Process capability





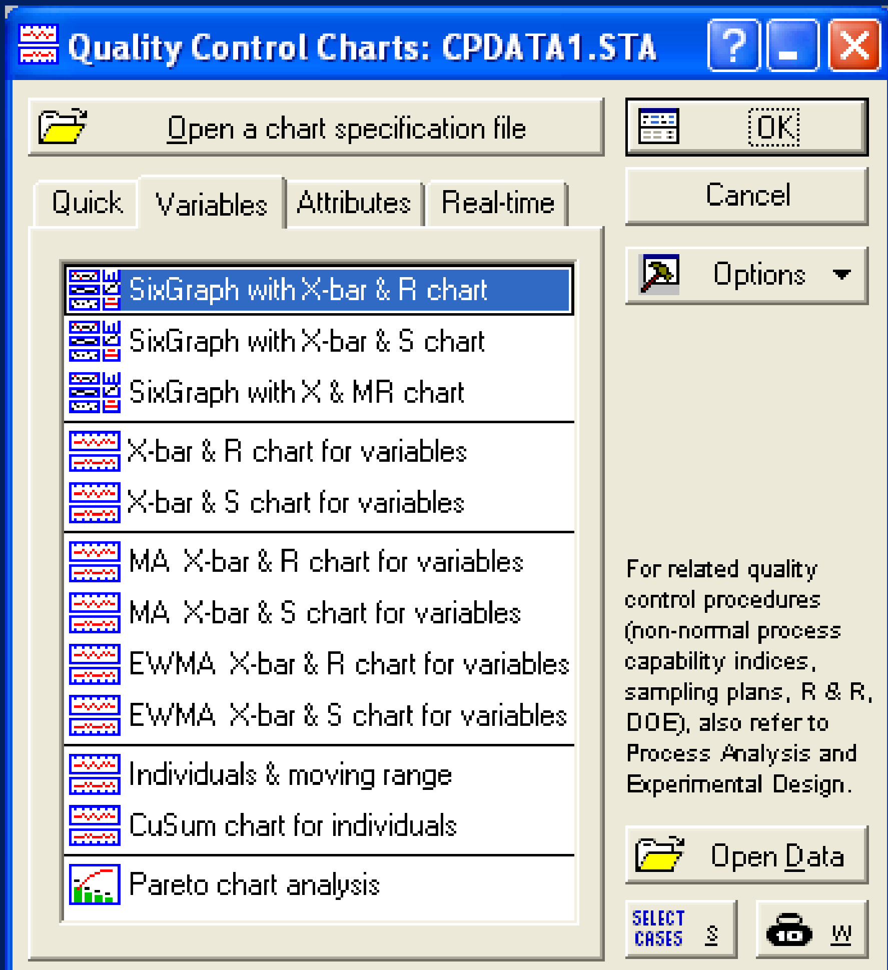


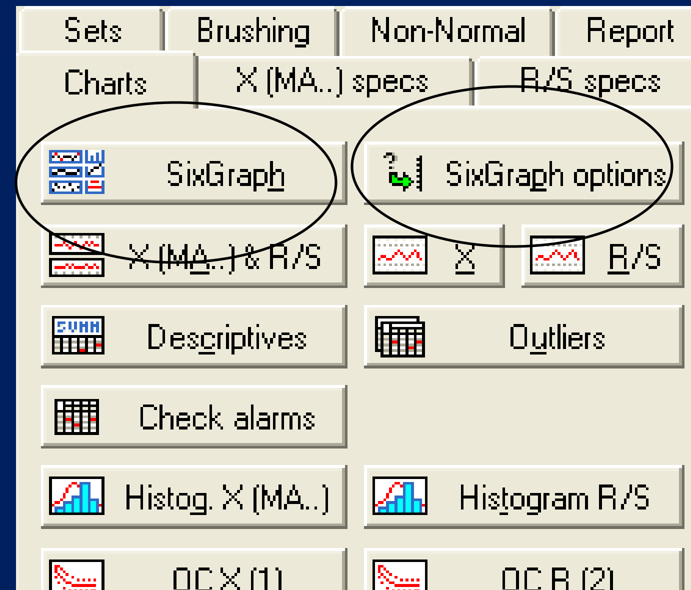
Capability Histogram: YS5
Sigma (Total):1.34984 Sigma (Within):1.0028
Within SD: 1.003; Cp: .3324; Cpk: .073
Overall SD: 1.350; Pp: .2469; Ppk: .054
LSL: 249.0; Nom.: 250.0; USL: 251.
-3.*S(T)
LSL
Nominal
USL
+3.*S(T)
20
15
10
5
0
245.5 246.5 247.5 248.5 249.5 250.5 251.5 252.5 253.5 254.5 255.5
246.0 247.0 248.0 249.0 250.0 251.0 252.0 253.0 254.0 255.0 256.0
Total
Within

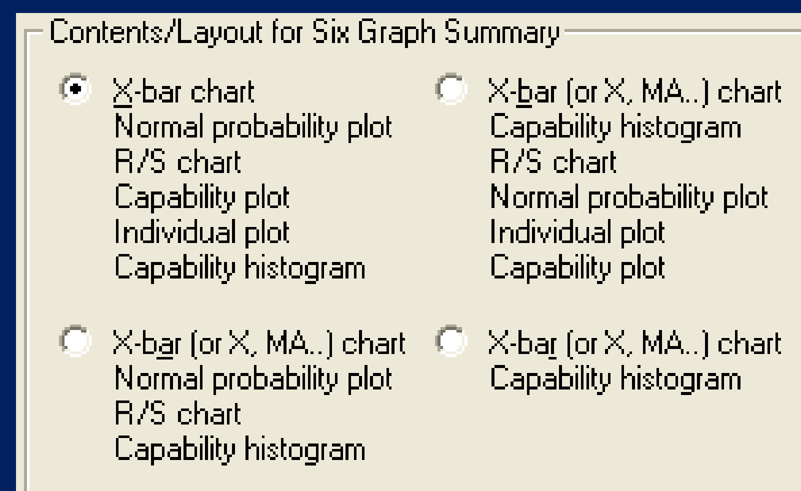


X-Bar/R: YS5: CPDATA1.STA
Sets
Brushing
Non-Normal
Report
Charts
X (MA..) specs
R/S specs
SixGraph
SixGraph options
X (MA..) & R/S
X
R/S
Descriptives
Outliers
Check alarms
Histog. X (MA..)
Histogram R/S
OC X (1)
OC R (2)
Runs tests
X non-normal
Process capability
Histogram
Summary
Options...
Save As...
Cancel
Brush...
Update
Lock...
By Group

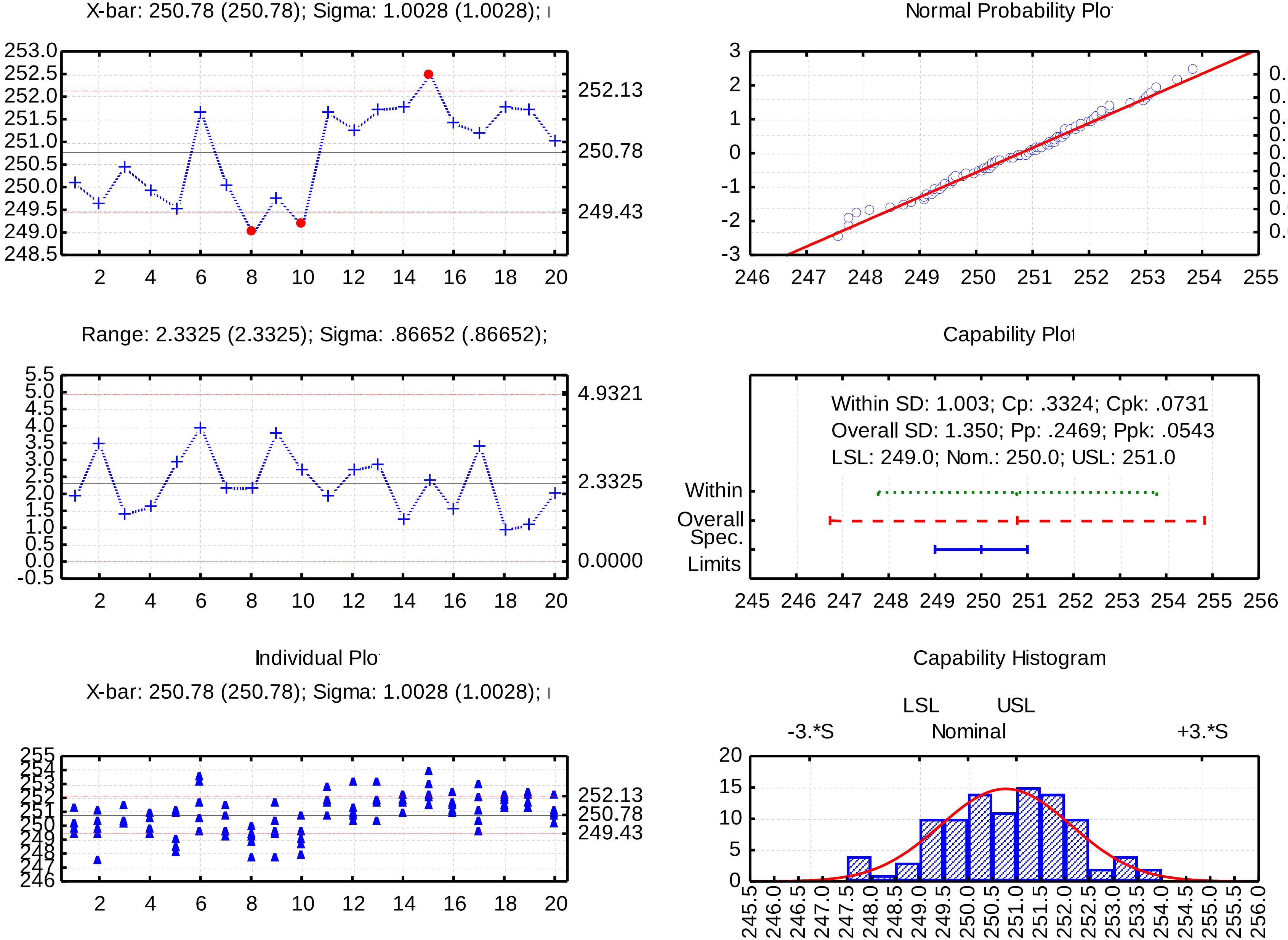
Sixgraph


Quality Control Charts: CPDATA1.STA
Open a chart specification file
OK
Cancel
Options
Quick
Variables
Attributes
Real-time
SixGraph with X-bar & R chart
SixGraph with X-bar & S chart
SixGraph with X & MR chart
X-bar & R chart for variables
X-bar & S chart for variables
MA X-bar & R chart for variables
MA X-bar & S chart for variables
EWMA X-bar & R chart for variables
EWMA X-bar & S chart for variables
Individuals & moving range
CuSum chart for individuals
Pareto chart analysis
For related quality control procedures (non-normal process capability indices, sampling plans, R & R, DOE), also refer to Process Analysis and Experimental Design.
Open Data
SELECT CASES
S
W


Sets
Brushing
Non-Normal
Report
Charts
X (MA..) specs
R/S specs
SixGraph
SixGraph options
X (MA..) & R/S
X
R/S
Descriptives
Outliers
Check alarms
Histog. X (MA..)
Histogram R/S
OC X (1)
OC R (2)


Contents/Layout for Six Graph Summary
X-bar chart
Normal probability plot
R/S chart
Capability plot
Individual plot
Capability histogram
X-bar (or X, MA..) chart
Capability histogram
R/S chart
Normal probability plot
Individual plot
Capability plot
X-bar (or X, MA..) chart
Normal probability plot
R/S chart
Capability histogram
X-bar (or X, MA..) chart
Capability histogram

SixGraph X-bar and R Chart: YS5



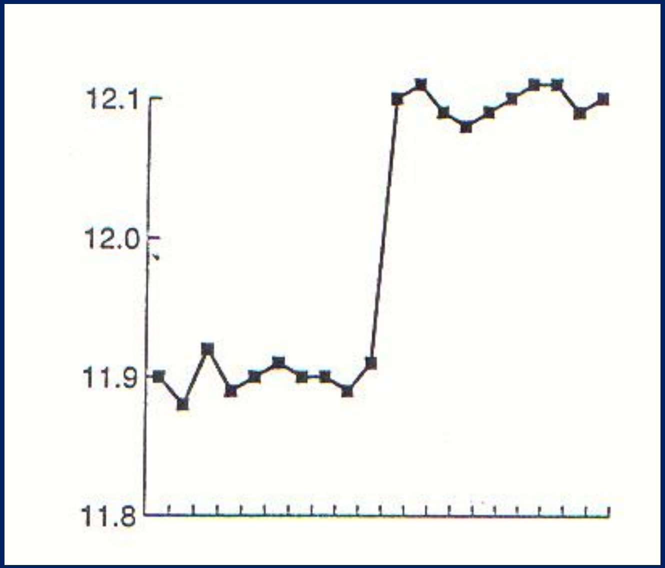
The use of plotting data

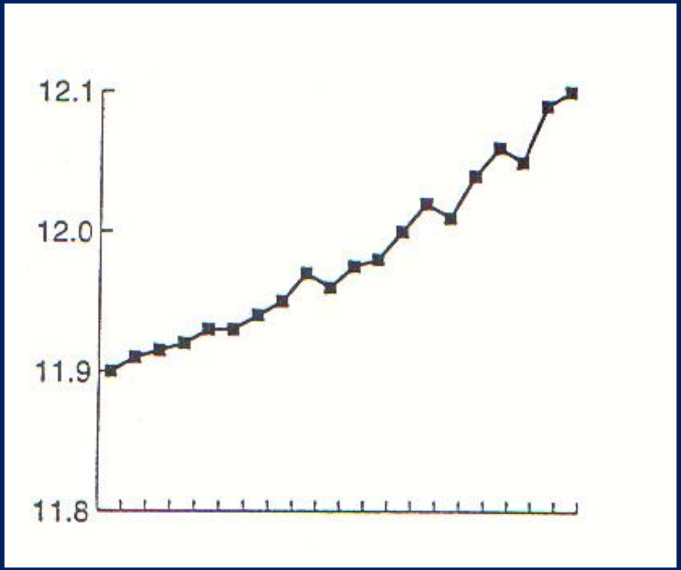
(T. Pyzdek: The Six Sigma Handbook, McGraw-Hill - Quality Publishing, 1999), p. 332

A sample of 100 bottles taken from a filling process has an average of 11.95 ounces, the standard deviation is 0.1 ounce

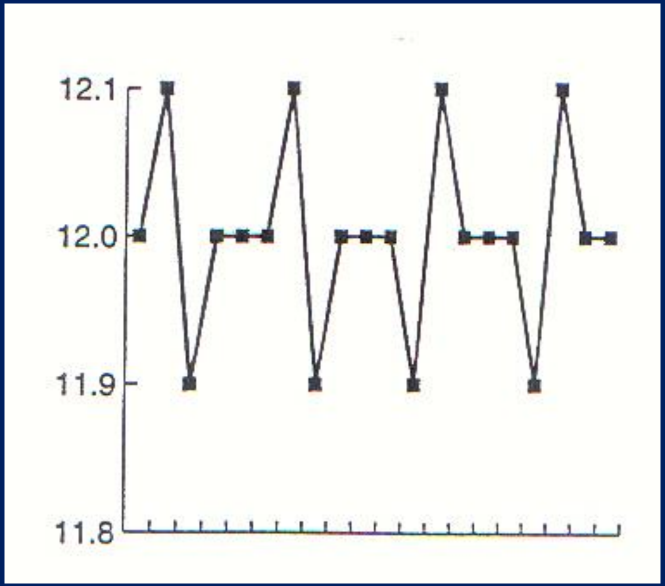
USL=12.1, LSL=11.9

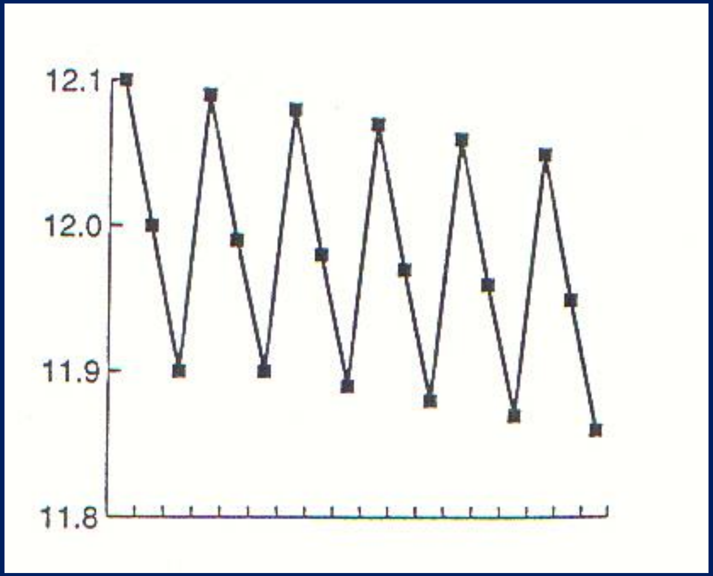
What to do with the process?





(run charts)





When to use X-bar chart

- if subgroups (at similar conditions) may be drawn from the process;
- if large ($\Delta \geq 2\sigma$) deviations are expected, and these are to be detected;
- if small deviations do not cause serious economic consequences;
- if the simplicity of the procedure is a point, but computation of sample mean is feasible;
- the cost of sampling is relatively low.

When not to use X-bar chart

- if subgroups (at similar conditions) may not be drawn from the process;
- if the within-groups fluctuation is much smaller than the between-groups fluctuation, since in this case many outliers were obtained;
- if the deviation to be detected is in the range $0.5\sigma < \Delta < 2\sigma$;
- if the cost of sampling/analysis is higher than could be gained by control;
- the process inherently cyclic or it contains trend, in that case the consecutive samples are not independent.

Steps for preparing and applying the X-bar/R chart

- Variable selection : relevant for quality, particular attention is paid to process elements which received high ranking in QFD, the measurement should not cost more than omitting the control.
- Deciding on rational subgroups (items produced under essentially the same conditions: the within-subgroup variation should be much less than the fluctuation between subgroups, when possible, consecutive units are used.
- Preliminary estimation of the fluctuation parameter for the process (σ^2) in order to decide the subgroup size; range is used for $n<10$. The subgroup size is usually 4-6, 5 is typical.

- Phase I: Data collection for estimating process parameters (μ and σ^2), usually 25 subgroups are taken. Plotting the data on charts (location and spread), computation of center line and control limits (trial control limits).
- Deciding on stability (control): If instability occurs, the special causes are found and eliminated. The belonging points are scratched, control limits are recalculated. This procedure is repeated until stability is achieved, additional samples may be drawn if required. This is the end of Phase I.

- On-going control (Phase II) is started if the process is proved to be in control. The analysis is started with the chart of fluctuation (e.g. range) because the control limits of the X-bar chart are valid only for σ=const case. If an outlier occurs, printing error is assumed first (its detection is cheap). The on-going control is to be performed real-time, it has not much sense to discover the necessity of an action for the previous day.

Control chart for individual values

It is not feasible to use averages and ranges:

• the production rate is too slow

• the output is too homogeneous over short time intervals (e.g. concentration of a solution).

Individual value (I or X) chart

Center line and control limits:

$$CL_x = \bar{x}$$

$$\overline{MR} = \frac{\sum_{i=2}^{m} MR_i}{m-1}$$

$$MR_i = |x_i - x_{i-1}|$$

(Moving Range)

$$UCL_x = \bar{x} + \frac{3\overline{MR}}{d_2}$$

$$\hat{\sigma} = \frac{\overline{MR}}{d_2}$$

$$LCL_x = \bar{x} - \frac{3\overline{MR}}{d_2}$$

Moving Range (MR) chart

Center line and control limits:

$$CL_{MR} = \overline{MR}$$

$$UCL_{MR} = D_4 \overline{MR}$$

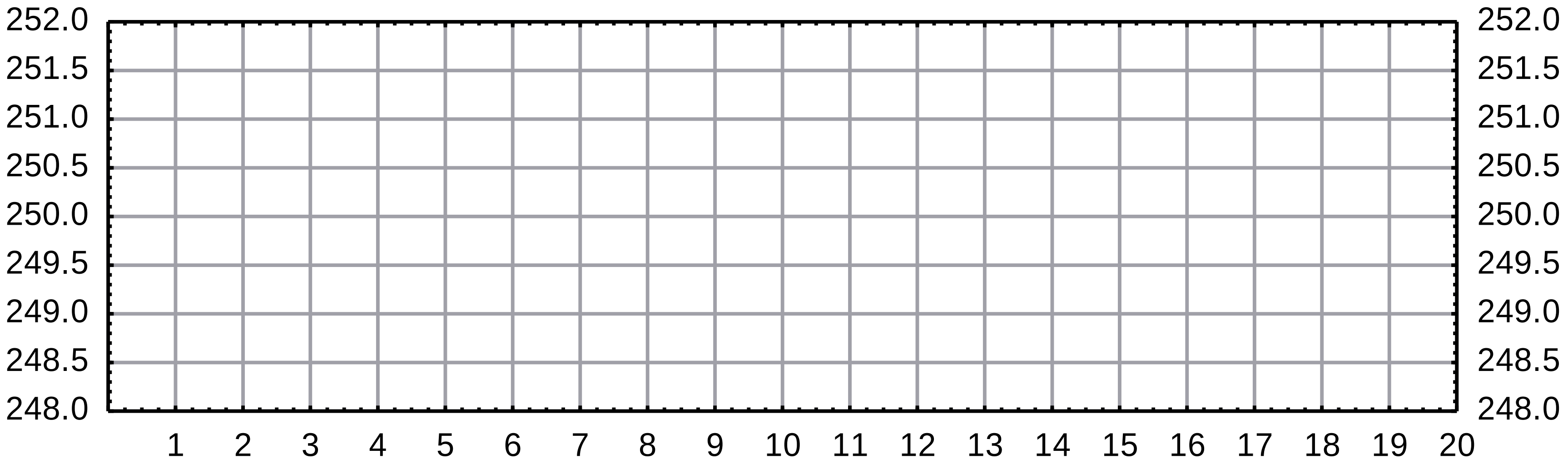
$$UCL_R = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3\frac{d_3\overline{R}}{d_2} = D_4\overline{R}$$

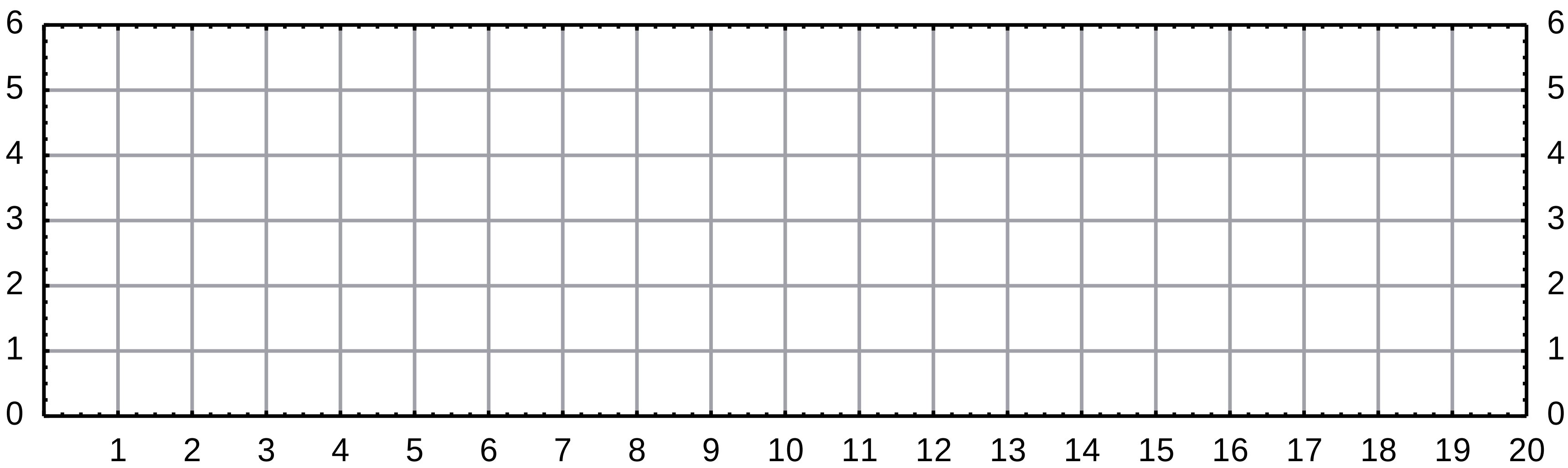
$$LCL_{MR} = D_3 \overline{MR}$$

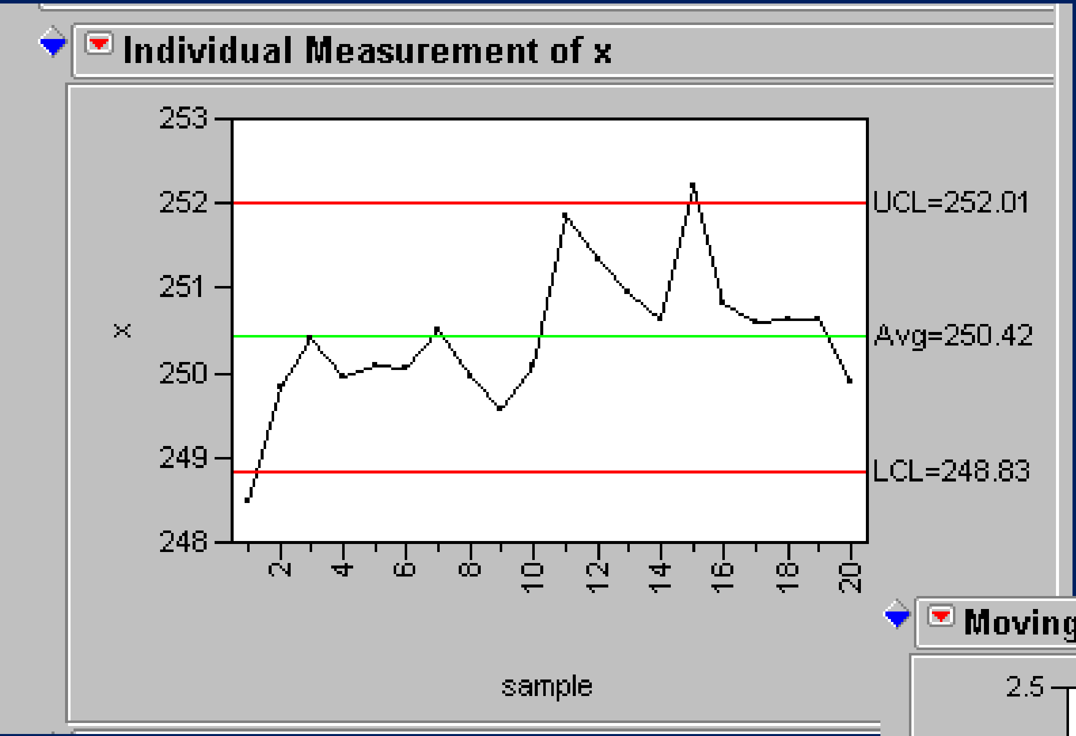
Example 19

Prepare an individual value + moving range chart from the data in the table!

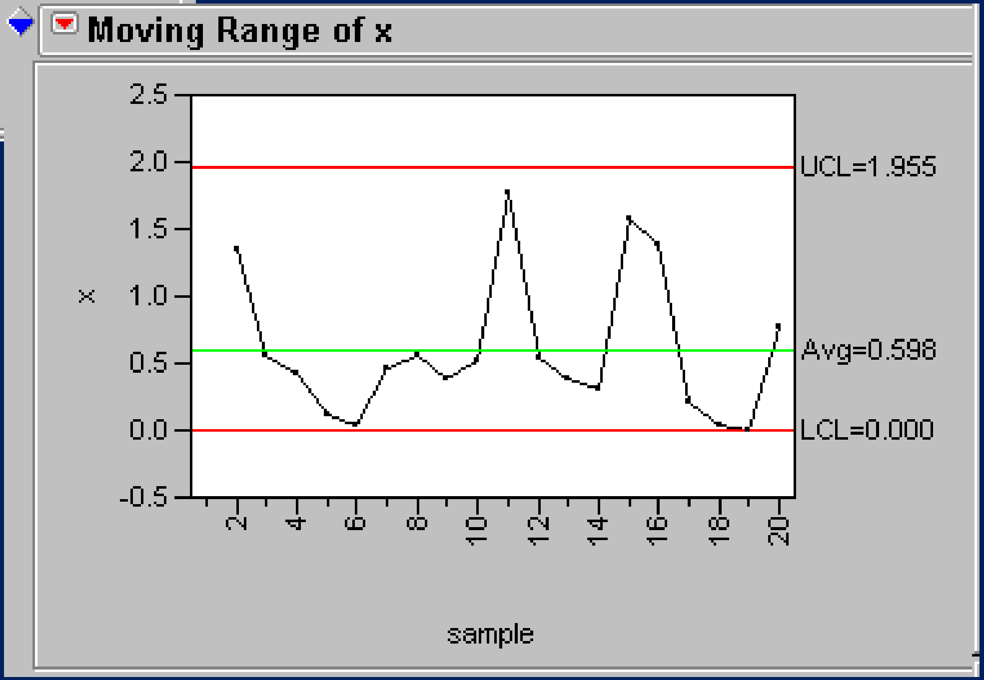
	x_i	$MR_i = \lvert x_i - x_{i-1} \rvert$
1	248.49	-
2	249.84	1.35
3	250.39	
4	249.96	
5	250.08	
6	250.04	
7	250.50	0.46
8	249.95	0.55
9	249.57	0.38
10	250.09	0.52
11	251.86	1.77
12	251.32	0.54
13	250.94	0.38
14	250.63	0.31
15	252.21	1.58
16	250.83	1.38
17	250.61	0.22
18	250.64	0.03
19	250.64	0.00
20	249.88	0.76
average	250.4235	0.5984


252.0
251.5
251.0
250.5
250.0
249.5
249.0
248.5
248.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20



Open Data Table: Indiv1.xls
Graph>ControlChart
Chart Type: IR
Process: YS
Sample Label: Sample

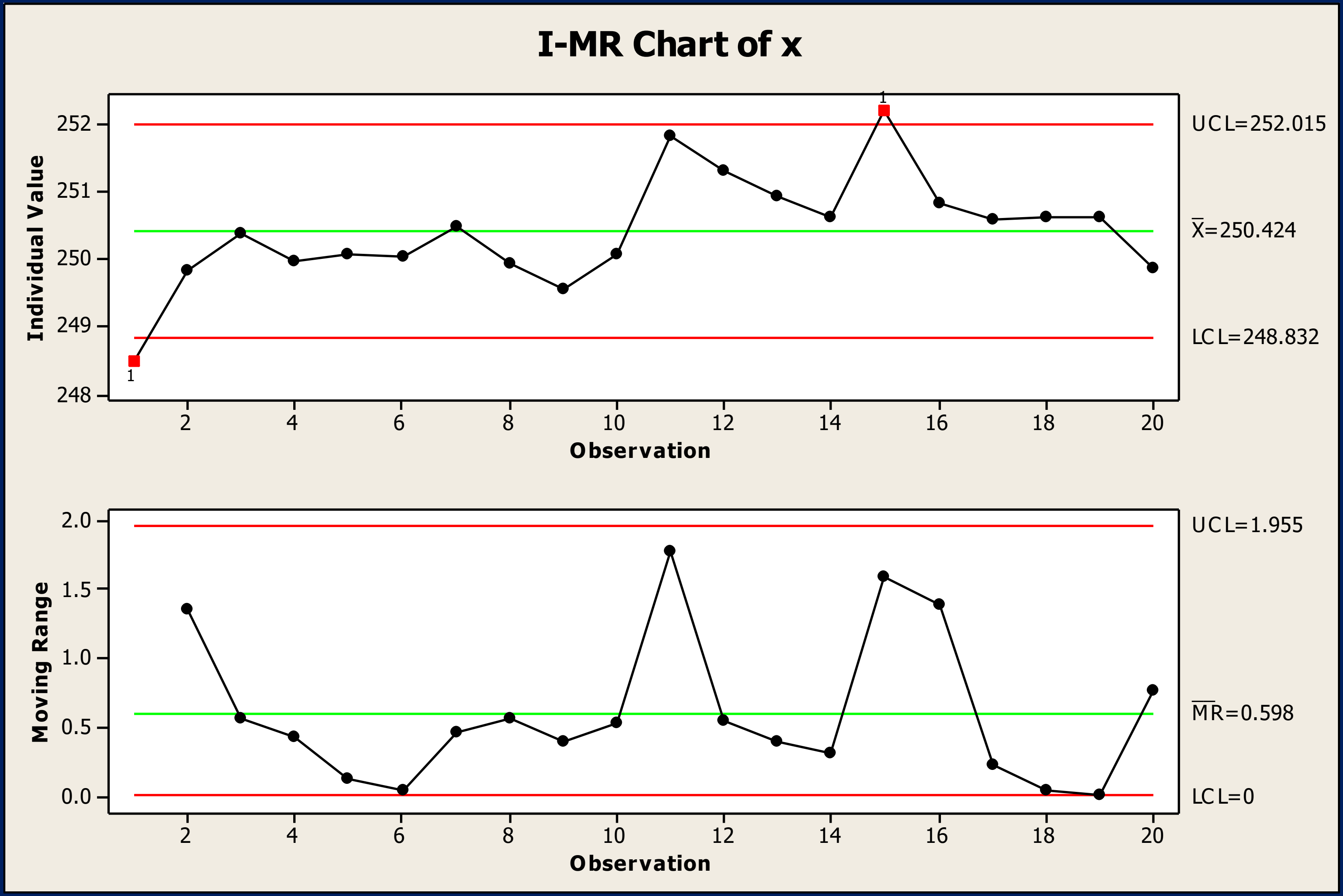


Example 19

Prepare an individual value + moving range chart from the data in the Indiv1.xls!

Phase I or Phase II?

Minitab>Stat>Control Charts>Variables Charts for Individuals>I-MR

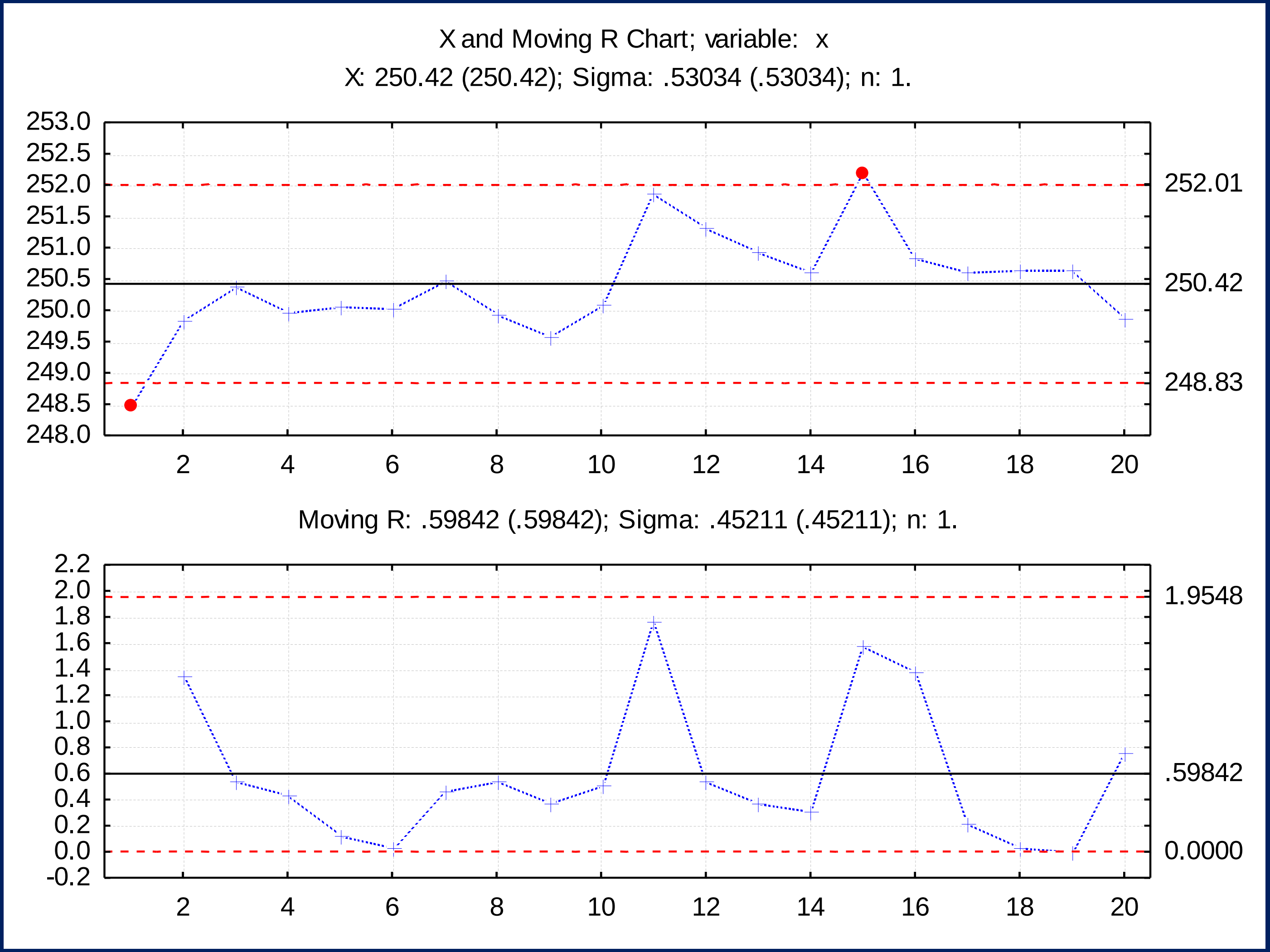

I-MR Chart of x
Individual Value
252
251
250
249
248
1
1
UCL=252.015
X̄=250.424
LCL=248.832
2
4
6
8
10
12
14
16
18
20
Observation
Moving Range
2.0
1.5
1.0
0.5
0.0
UCL=1.955
M̄R=0.598
LCL=0
Observation

Example 14

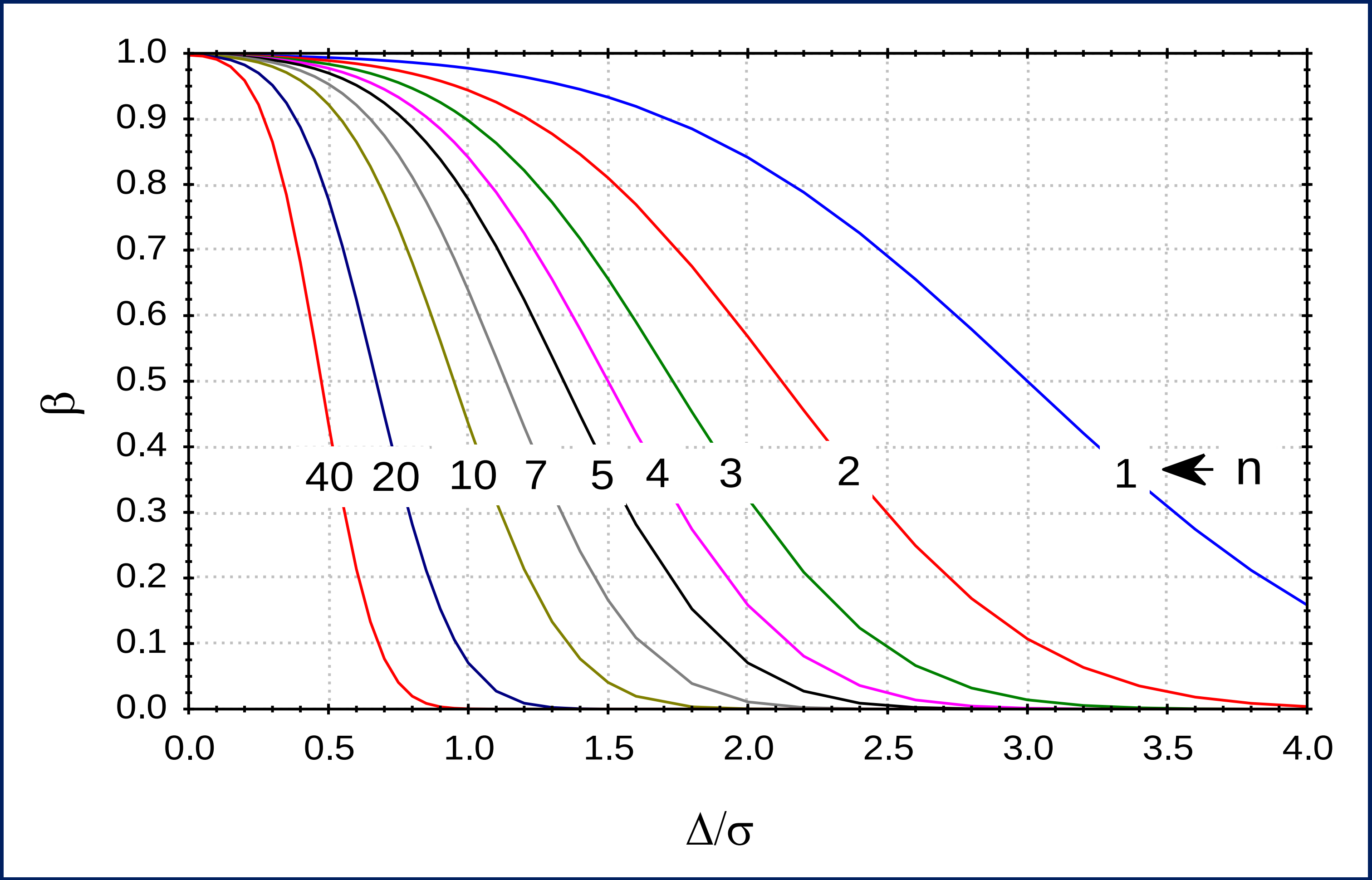
Prepare an individual value + moving range chart from the data in the Indiv1.xls!

Phase I or Phase II?

Statistics>Industrial Statistics>Quality Control Charts
Individuals & moving mange
Variables: X


X and Moving R Chart; variable: x
X: 250.42 (250.42); Sigma: .53034 (.53034); n: 1.
253.0
252.5
252.0
251.5
251.0
250.5
250.0
249.5
249.0
248.5
248.0
252.01
250.42
248.83
2
4
6
8
10
12
14
16
18
20
Moving R: .59842 (.59842); Sigma: .45211 (.45211); n: 1.
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1.9548
.59842
0.0000
2
4
6
8
10
12
14
16
18
20

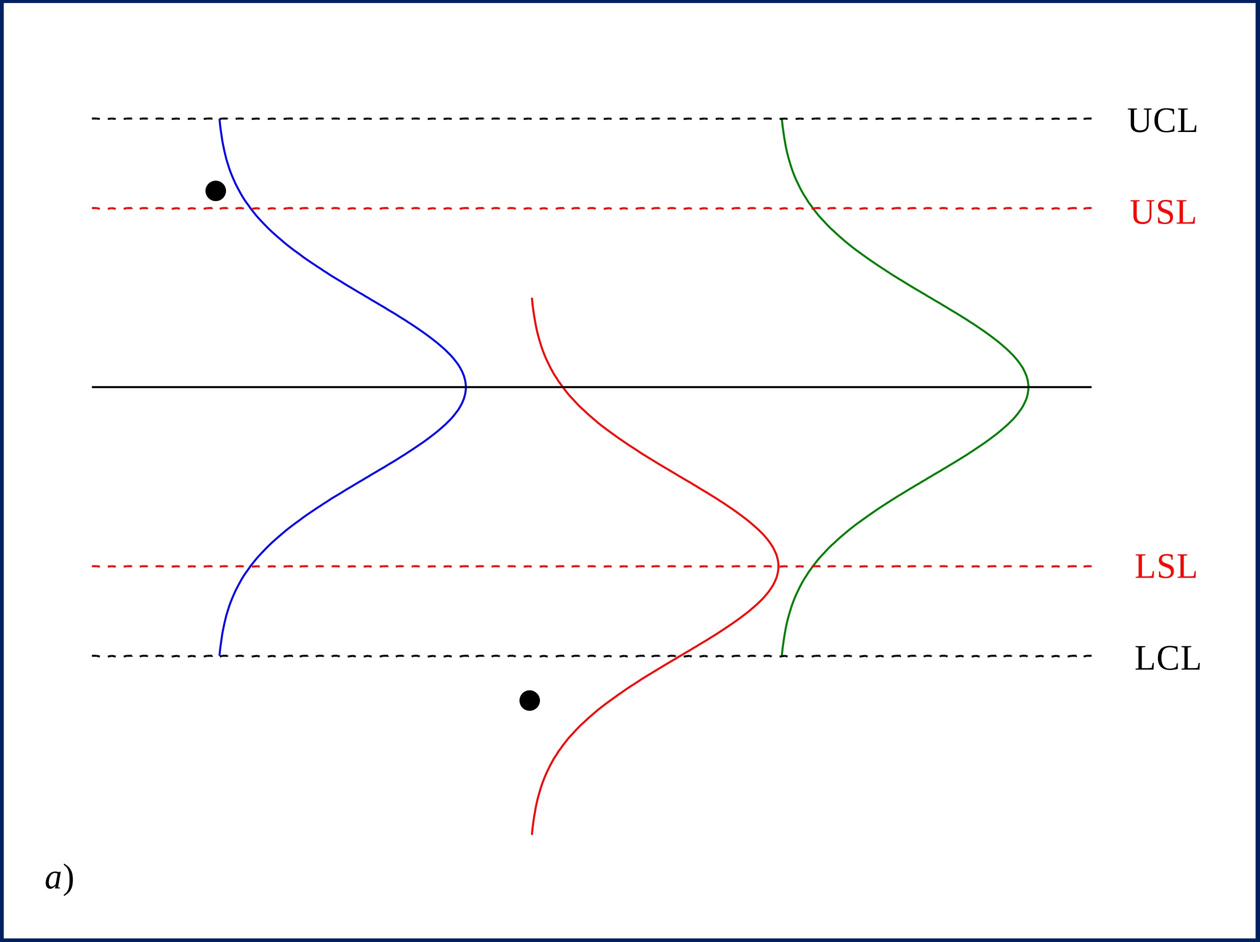
Operating Characteristic (OC) curve for the X-bar chart (α=0.0027)

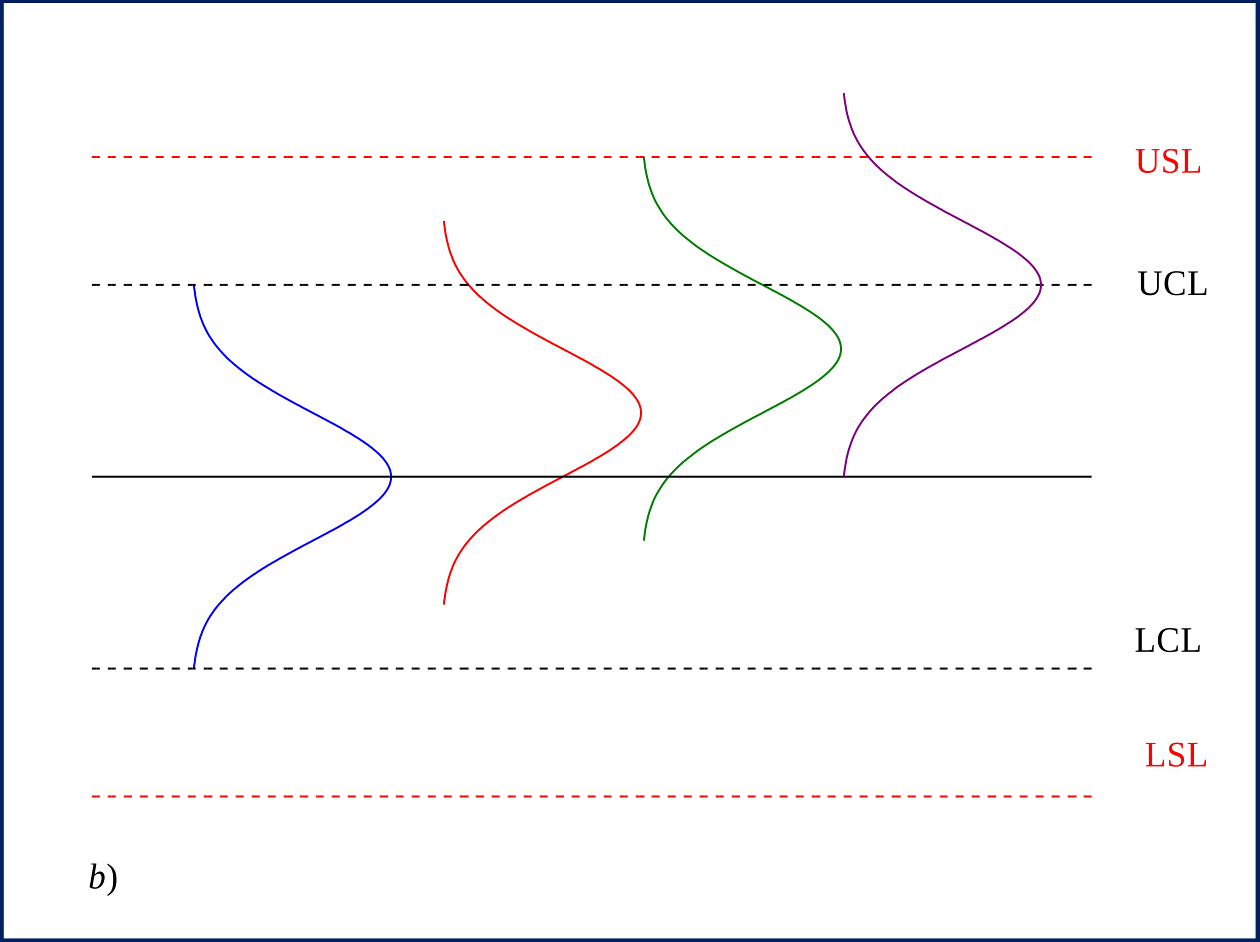


Summary table for the variables control charts

Type of the chart			
$\bar{x}-R$	$\bar{x}-s$	$\bar{x}-s^2$	x–MR
$CL_{\bar{x}}=\bar{\bar{x}}$	$CL_{\bar{x}}=\bar{\bar{x}}$	$CL_{\bar{x}}=\bar{\bar{x}}$	$CL_{x}=\bar{x}$
$UCL_{\bar{x}}=\bar{\bar{x}}+\frac{3\bar{R}}{d_2\sqrt{n}}=\bar{\bar{x}}+A_2\bar{R}$	$UCL_{\bar{x}}=\bar{\bar{x}}+3\frac{\bar{s}}{c_4\sqrt{n}}=\bar{\bar{x}}+A_3\bar{s}$	$UCL_{\bar{x}}=\bar{\bar{x}}+3\frac{\sqrt{\overline{s^2}}}{\sqrt{n}}$	$UCL_{x}=\bar{x}+\frac{3\overline{MR}}{d_2}$
$LCL_{\bar{x}}=\bar{\bar{x}}-\frac{3\bar{R}}{d_2\sqrt{n}}=\bar{\bar{x}}-A_2\bar{R}$	$LCL_{\bar{x}}=\bar{\bar{x}}-3\frac{\bar{s}}{c_4\sqrt{n}}=\bar{\bar{x}}-A_3\bar{s}$	$LCL_{\bar{x}}=\bar{\bar{x}}-3\frac{\sqrt{\overline{s^2}}}{\sqrt{n}}$	$LCL_{x}=\bar{x}-\frac{3\overline{MR}}{d_2}$
$CL_R=\bar{R}$	$CL_s=\bar{s}$	$CL_{s^2}=\overline{s^2}$	$CL_{MR}=\overline{MR}$
$UCL_R=\bar{R}+3\frac{d_3\bar{R}}{d_2}=D_4\bar{R}$	$UCL_s=\bar{s}+3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2}=B_4\bar{s}$	$UCL_{s^2}=\frac{\overline{s^2}\chi^2_{fölsô}}{\nu}$	$UCL_{MR}=D_4\overline{MR}$
$LCL_R=\bar{R}-3\frac{d_3\bar{R}}{d_2}=D_3\bar{R}$	$LCL_s=\bar{s}-3\frac{\bar{s}}{c_4}\sqrt{1-c_4^2}=B_3\bar{s}$	$LCL_{s^2}=\frac{\overline{s^2}\chi^2_{alsó}}{\nu}$	$LCL_{MR}=D_3\overline{MR}$

Why not the specification limits are used in the chart?




USL
UCL
LCL
LSL
b)

Multiple stream (group) control charts

For multiple-stream processes (e.g., operators, machines, assembly lines); summarising the measurements for all streams simultaneously.

Example 31

An automatic filling machine with 8 heads are used to fill mustard to bottles.

Prepare a control chart for Phase I!

mustard.sta

The samples from the 8 heads are not elements of a single process, they mean 8 different processes

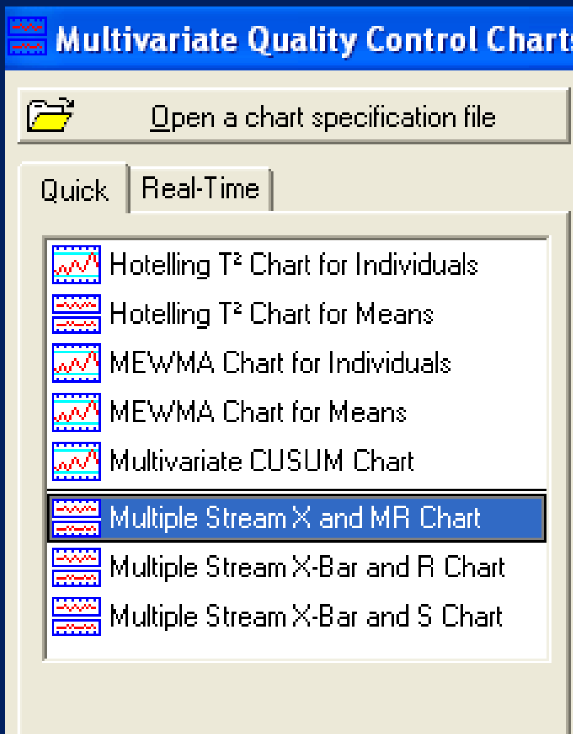
8 I-MR charts

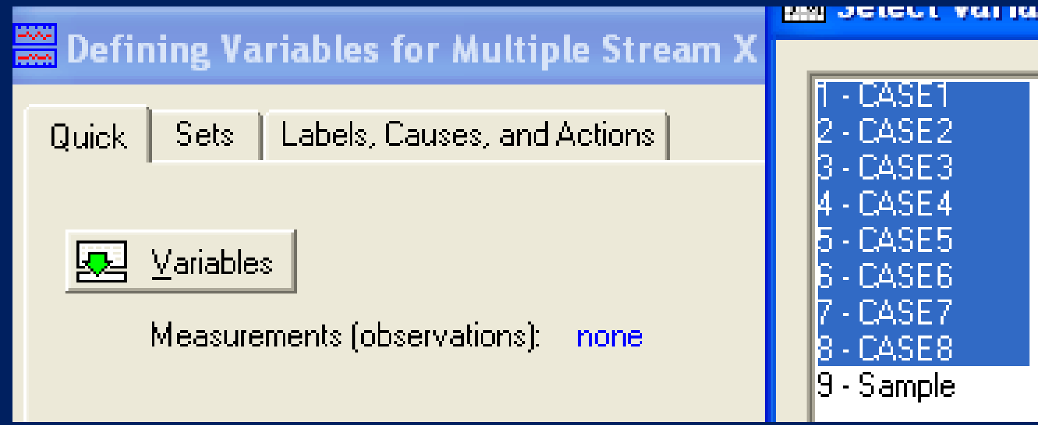
From among the values (means and ranges) the smallest and largest are plotted only.

If these extreme values are within the control limits, the rest are there as well.

sample	HEAD1	HEAD2	HEAD3	HEAD4	HEAD5	HEAD6	HEAD7	HEAD8
1	378	375	367	370	384	372	372	371
2	376	372	362	367	383	373	370	379
3	372	385	373	372	386	380	374	376
4	379	375	370	371	385	380	374	375
5	374	373	362	380	383	372	370	368
6	352	371	366	370	385	371	377	378
7	370	377	370	374	385	380	370	370
8	377	379	367	370	385	372	367	372
9	370	380	367	373	383	369	373	371
10	369	374	366	375	383	370	379	369
11	373	376	374	373	388	372	371	378
12	375	380	371	377	388	368	376	371
13	380	375	374	376	386	380	376	370
14	372	373	375	383	387	378	375	376
15	380	375	370	374	386	368	373	376
16	379	372	373	372	386	378	368	374
17	372	376	369	373	388	381	376	371
18	368	372	372	375	387	380	380	375
19	372	370	370	375	386	379	375	371
20	371	375	383	383	380	379	377	382
21	370	376	380	376	386	374	375	380
22	376	373	368	374	386	370	375	380
23	372	373	372	379	385	381	380	375
24	375	372	369	370	386	372	379	375
25	383	380	369	370	386	375	375	373

Statistics>Industrial Statistics & Six Sigma>Multivariate Quality Control>Multiple Stream X and MR Chart





filling machine with 8 heads