MEASUREMENT SYSTEM ANALYSIS

(R&R study)

R&R study

The purpose is to check if the error in measurement system is small enough to get reliable data from the process studied.

Variables data
(interval and proportional scale: ^{0}C, kg, N)

Attribute data
(nominal and ordinal scale: good/bad, stage, rank)

Variables data

bias (accuracy)

precision (R&R)

- repeatability
- reproducibility by different operators
- ratio of precision (measurement error) to the variation between parts
- estimation of variance components

Accuracy (bias)

$E(x) = x_{ref}$ x_{ref} : standard

$H_0 : E(x) = x_{ref}$

one-sample t test $t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

H_0 (no bias) is accepted at α significance level if

$$P\left(-t_{a/2} < \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \le t_{a/2} \right) = 1 - \alpha$$

Example 15 (ext.8)

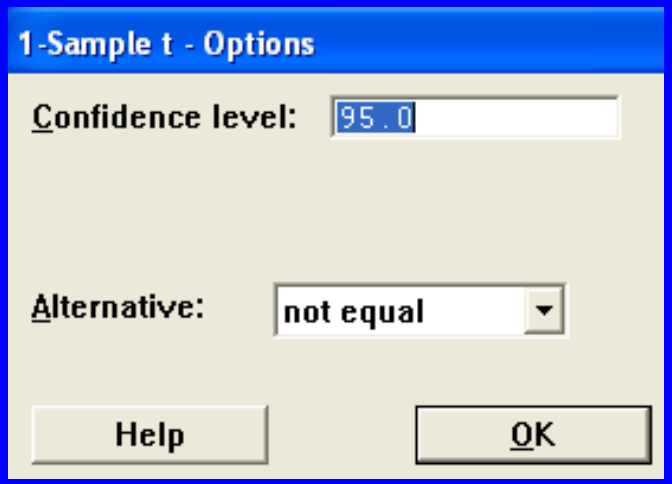
$H_0 : E(x) = x_{ref}$

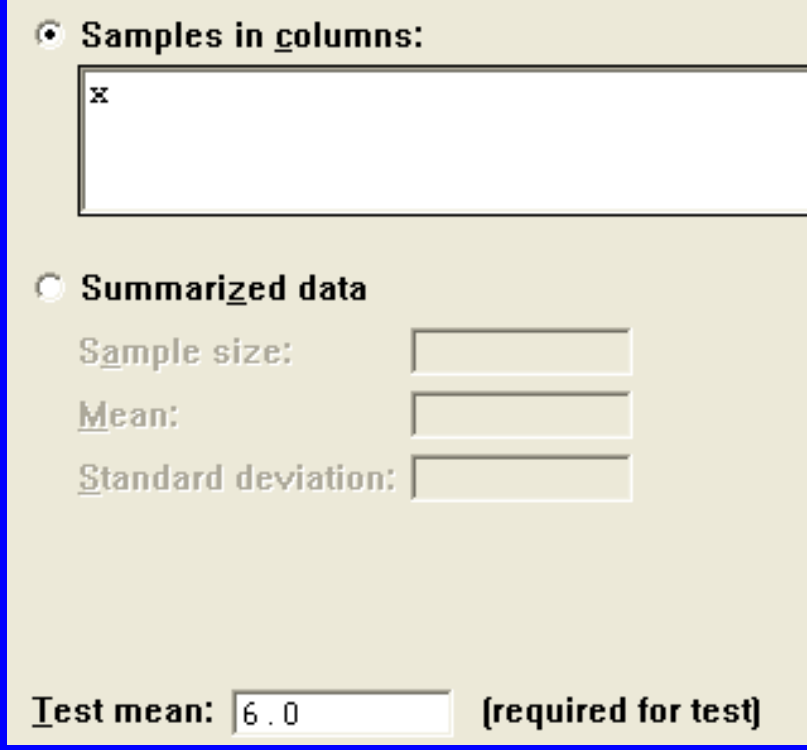
X_{ref} = 6.0 (standard)

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Minitab>Stat>Basic Statistics>1-sample t

i	x_i	$x_i - x_{ref}$
1	5.8	-0.2
2	5.7	-0.3
3	5.9	-0.1
4	5.9	-0.1
5	6.0	0.0
6	6.1	0.1
7	6.0	0.0
8	6.1	0.0
9	6.4	0.4
10	6.3	0.3
11	6.0	0.0
12	6.1	0.1
13	6.2	0.2
14	5.6	-0.4
15	6.0	0.0





Session

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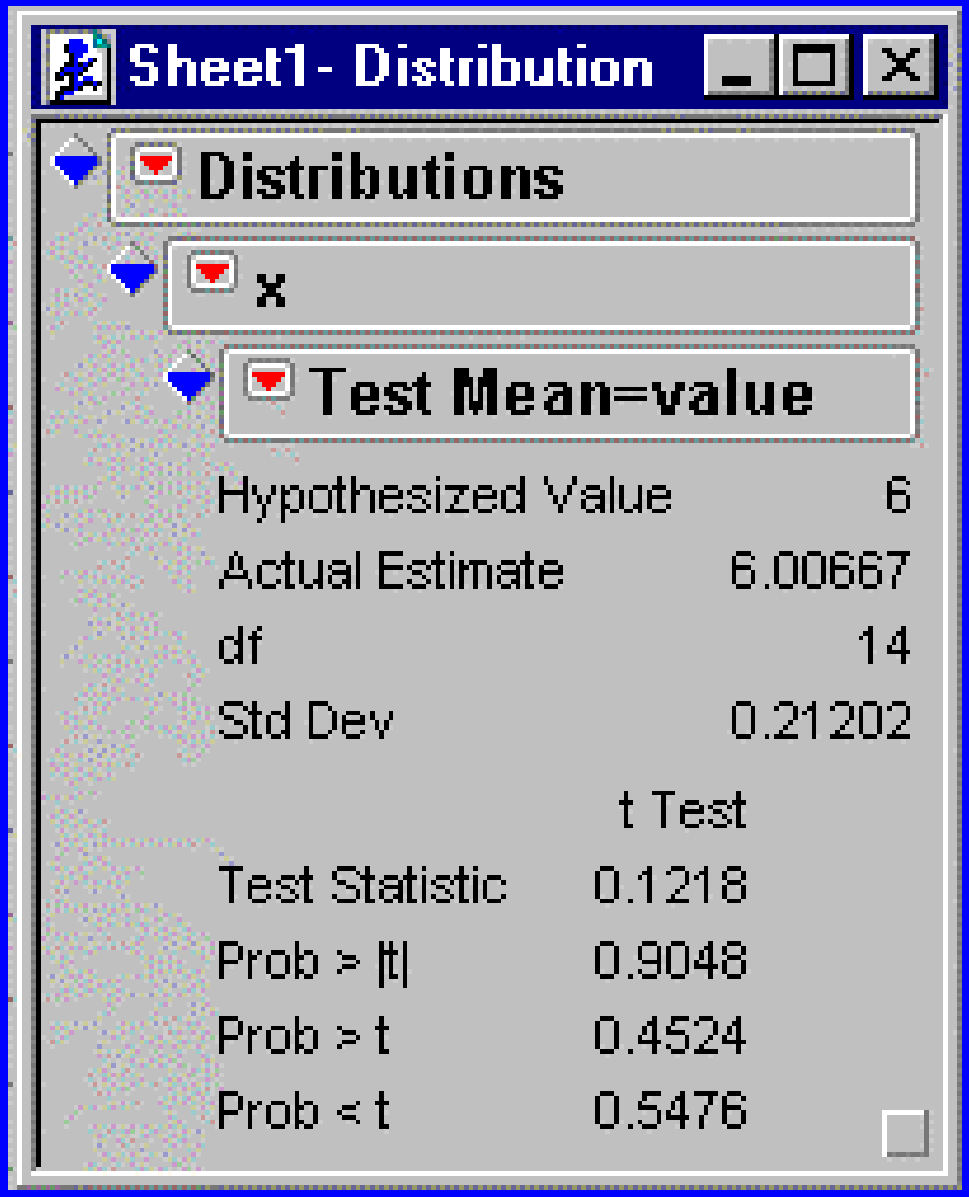
Worksheet was saved on 08/08/2004 22:19:06

Results for: Gagebias.MTW

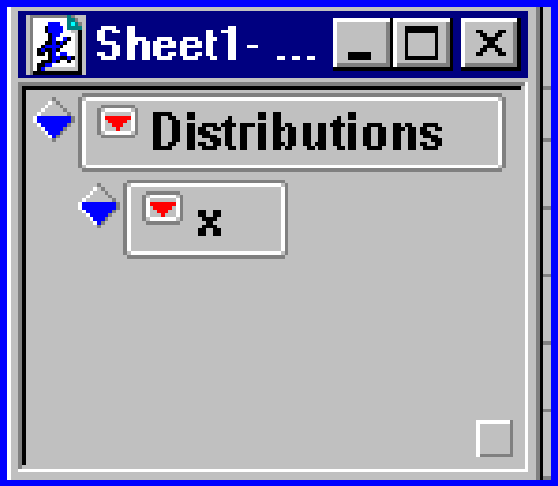
One-Sample T: x

Test of mu = 6 vs not = 6


Variable   N     Mean    StDev  SE Mean         95% CI           T      P
x         15  6.00667  0.21202  0.05474  (5.88925, 6.12408)  0.12  0.905
```

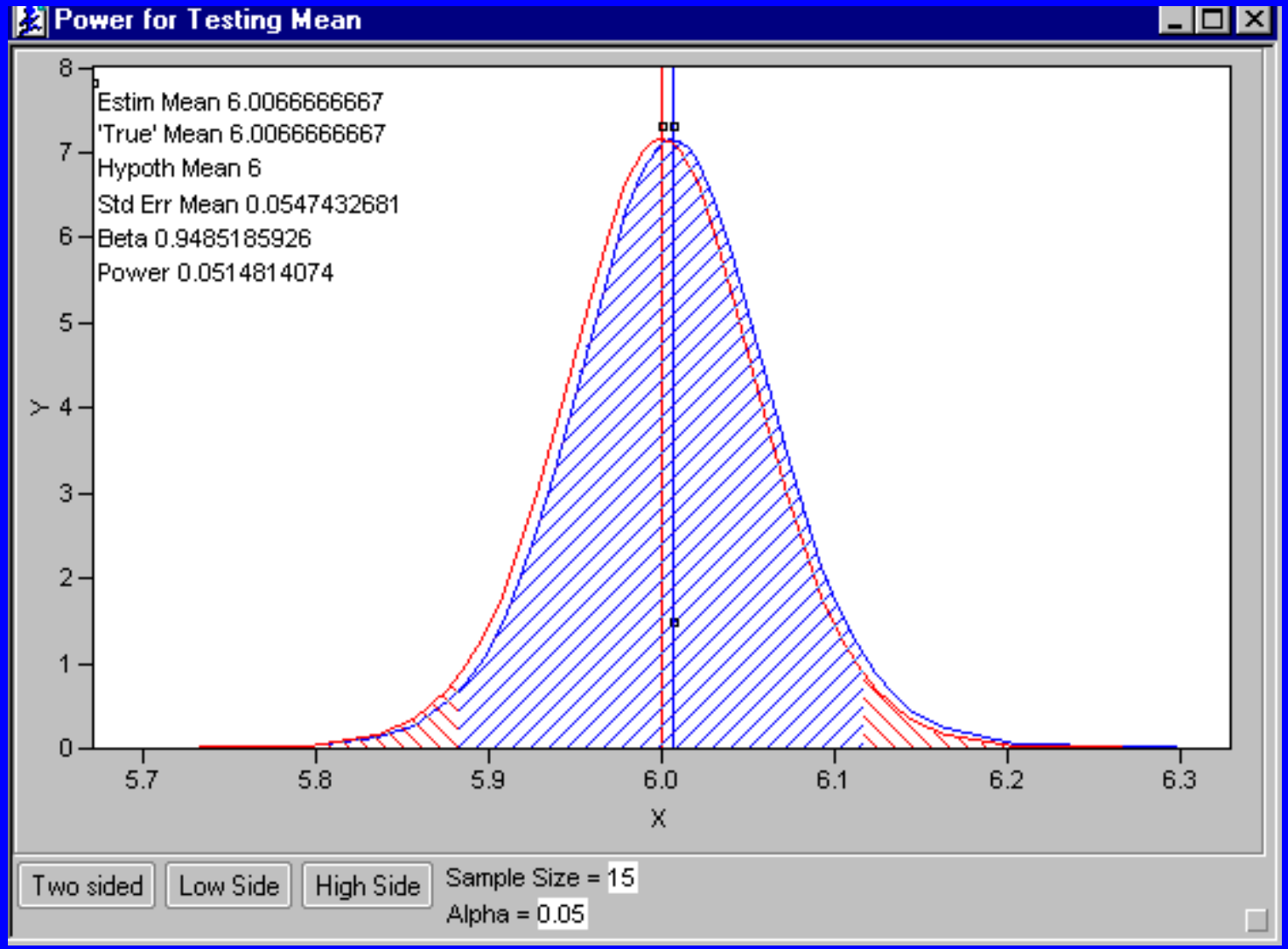


Open Data Table
gagebias.xls
Analyze>Distribution
Y, Column: x

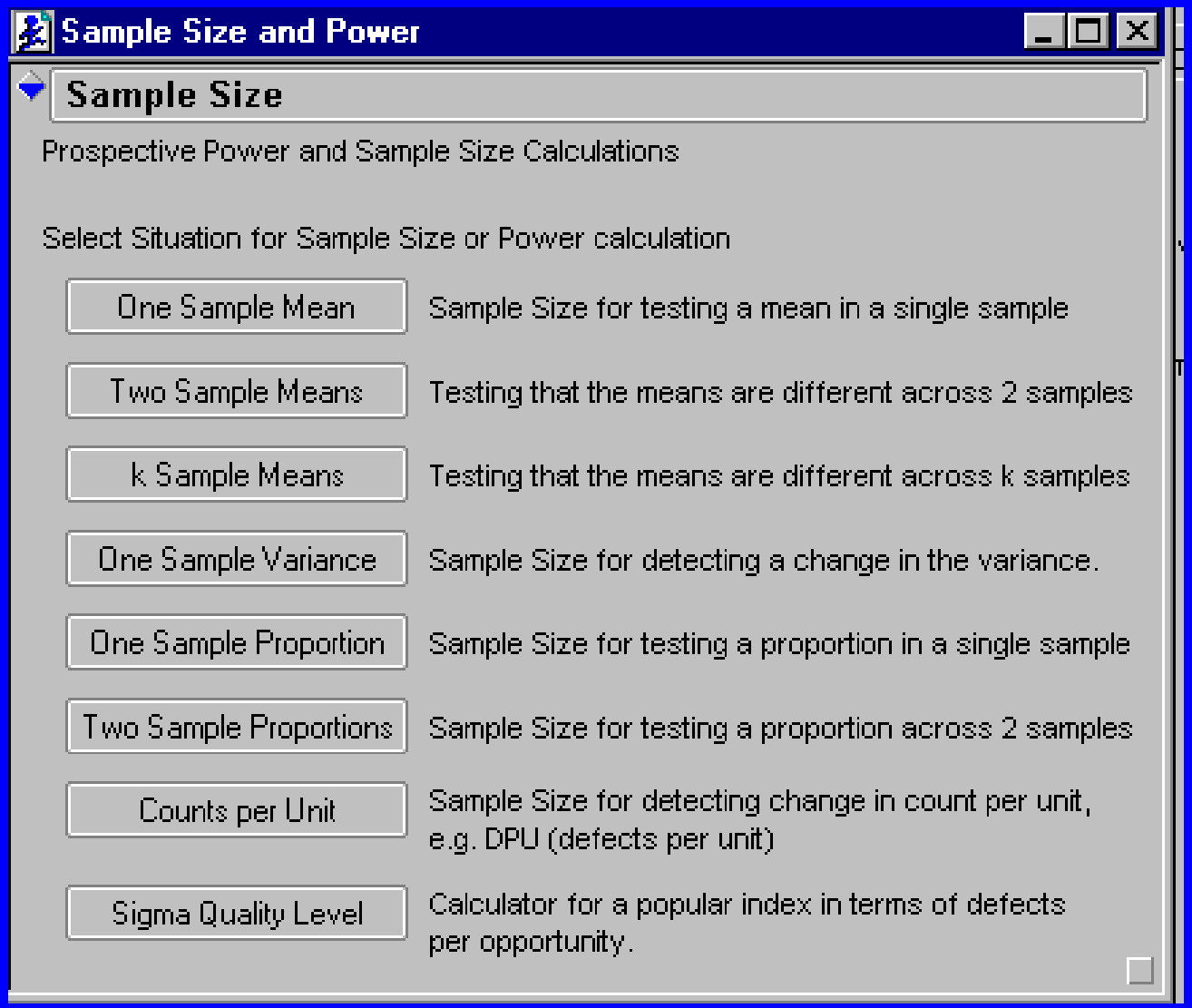


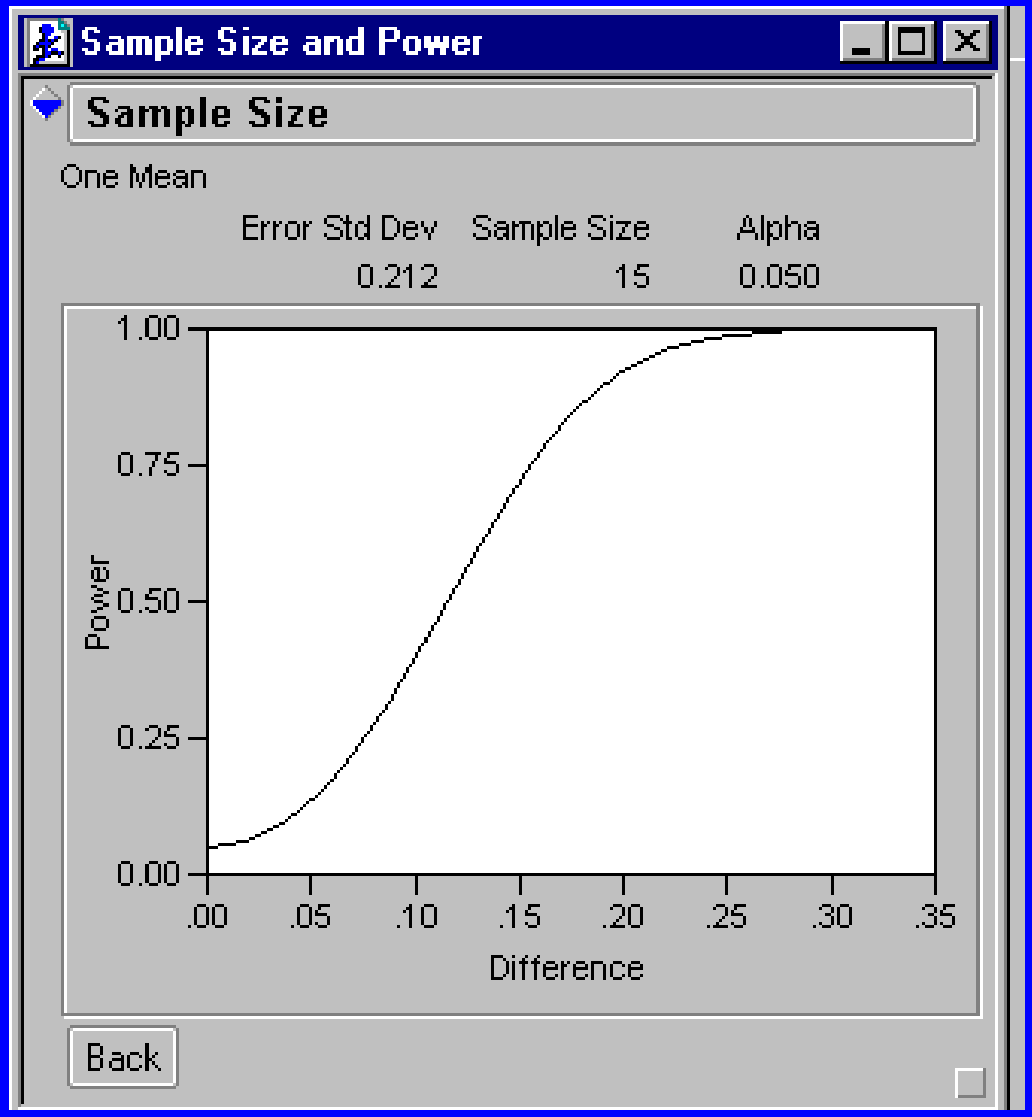
x: Test Mean

x: Test Mean
Power Animation



DOE>Sample size and power




Sample Size and Power
Sample Size
One Mean
Error Std Dev
Sample Size
Alpha
0.212
15
0.050
1.00
0.75
0.50
0.25
0.00
Power
.00
.05
.10
.15
.20
.25
.30
.35
Difference
Back

Example 15

$H_0 : E(x) = x_{ref}$ x_{ref}=6.0 (standard)

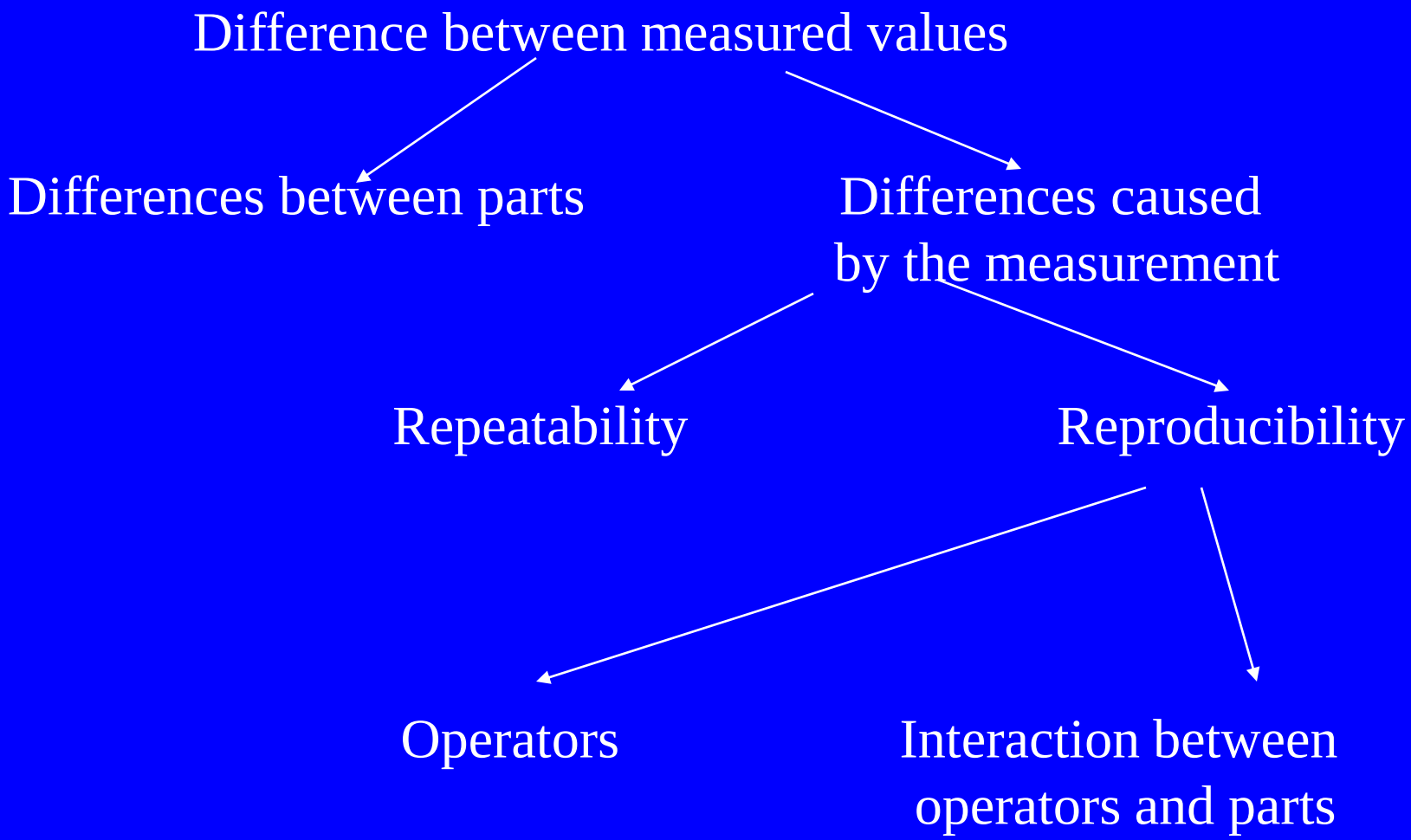
$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Statistics>Basic Statistics and Tables>
t-test, single sample

i	x_i	$x_i - x_{ref}$
1	5.8	-0.2
2	5.7	-0.3
3	5.9	-0.1
4	5.9	-0.1
5	6.0	0.0
6	6.1	0.1
7	6.0	0.0
8	6.1	0.0
9	6.4	0.4
10	6.3	0.3
11	6.0	0.0
12	6.1	0.1
13	6.2	0.2
14	5.6	-0.4
15	6.0	0.0

Variable	Test of means against reference constant (value) (gagebias)							
	Mean	Std.Dv.	N	Std.Err.	Reference Constant	t-value	df	p
x	6.006667	0.212020	15	0.054743	6.000000	0.121781	14	0.904804

Splitting the differences into components



Total variance of measurement data :

$$\sigma^2_{\text{total}} = \sigma^2_{\text{parts}} + \sigma^2_{\text{R\&R}}$$

Fluctuation attributable to the measurement (precision):

$$\sigma^2_{\text{R\&R}} = \sigma^2_{\text{reprod}} + \sigma^2_{\text{repeat}}$$

Reproducibility:

$$\sigma^2_{\text{reprod}} = \sigma^2_{\text{oper}} + \sigma^2_{\text{part*oper}}$$

Design of experiments for the study

A certain number (e.g. 10) is selected randomly from among the parts produced by the process to be investigated, all of them measured several (e.g. 3) times by each of the selected operators (e.g. 4).

operator	A			B			C		
part	rept 1	rept 2	rept 3	rept 1	rept 2	rept 3	rept 1	rept 2	rept 3
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									

Results:

- The variance components are related to the total variance.
- Analogously to the C_P process capability index the ranges attributed to the variance components is related to the width of the spec. range (P/T precision to tolerance) . Actually the 99% (5.15 σ width) interval is in the numerator:

$$\frac{P}{T} = \frac{5.15 \cdot \hat{\sigma}_{\text{R\&R}}}{USL - LSL}.$$

6.0 may stand for 5.15, expressing the ±3σ limit (99.73% instead of 99%)

Results:

- The variance components are related to the total variance.
- Analogously to the C_P process capability index the ranges attributed to the variance components is related to the width of the spec. range (P/T precision to tolerance) . Actually the 9.73% (6 σ width) interval is in the numerator:

$$\frac{P}{T} = \frac{6 \cdot \hat{\sigma}_{\text{R\&R}}}{USL - LSL}.$$

5.15 may stand for 6.0, expressing the 99% limit (instead of ±3s corresponding to 99.73%)

Number of distinguishable categories (discrimination index)

$$\frac{\hat{\sigma}_{\text{part}}}{\hat{\sigma}_{\text{R\&R}}}\sqrt{2}$$

rounded down to integer

Variance estimation: Range method

Variances are estimated from ranges, e.g.

$$\hat{\sigma}_{\text{repeat}} = \frac{\overline{R}_{\text{repeat}}}{d_2}$$

$\overline{R}_{\text{repeat}}$ is the average range of repetitions

d_2 is taken from a Table for the # of repetitions

Similarly for $\hat{\sigma}_{\text{reprod}}$ $\hat{\sigma}_{\text{part}}$

for small sample sizes different d_2 values apply

Variance estimation: ANOVA method

The model (two-way cross-classification with random factors, repeated measurements)

$$x_{ijk} = \mu + P_i + O_j + PO_{ij} + \varepsilon_{k(ij)}$$

P is for parts
O is for operators
ε experimental error

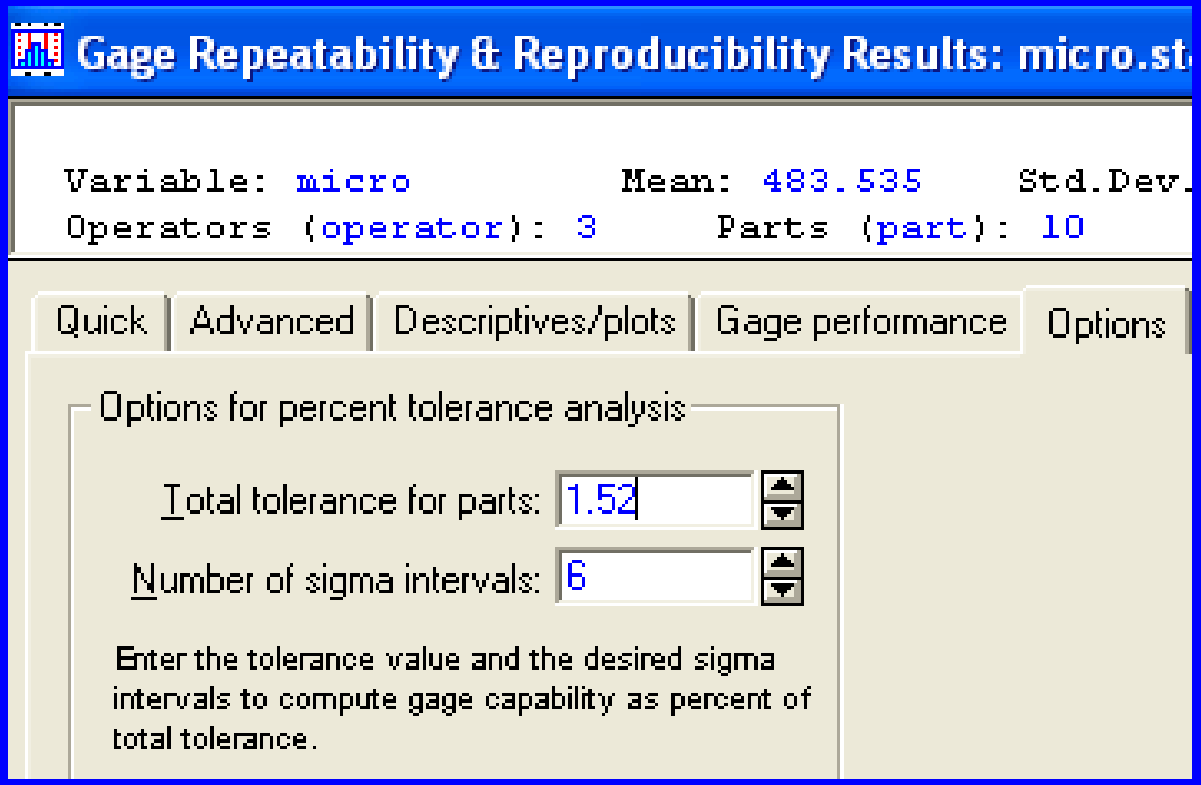
Example 24

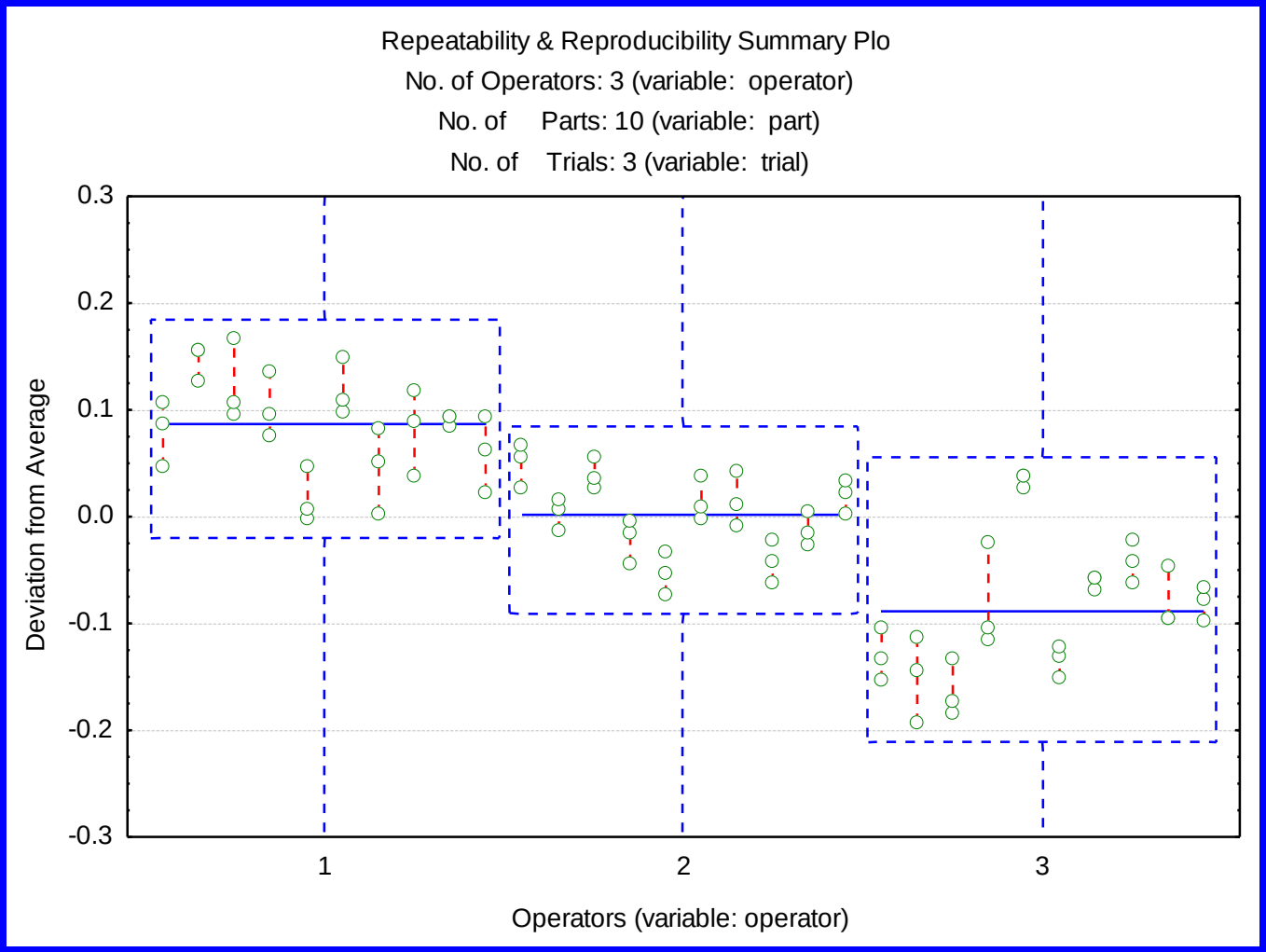
The width of the specification for the inner diameter 1.52 mm.
10 parts are taken randomly from the manufacturing, each of them are measured 3 times by 2 operators.

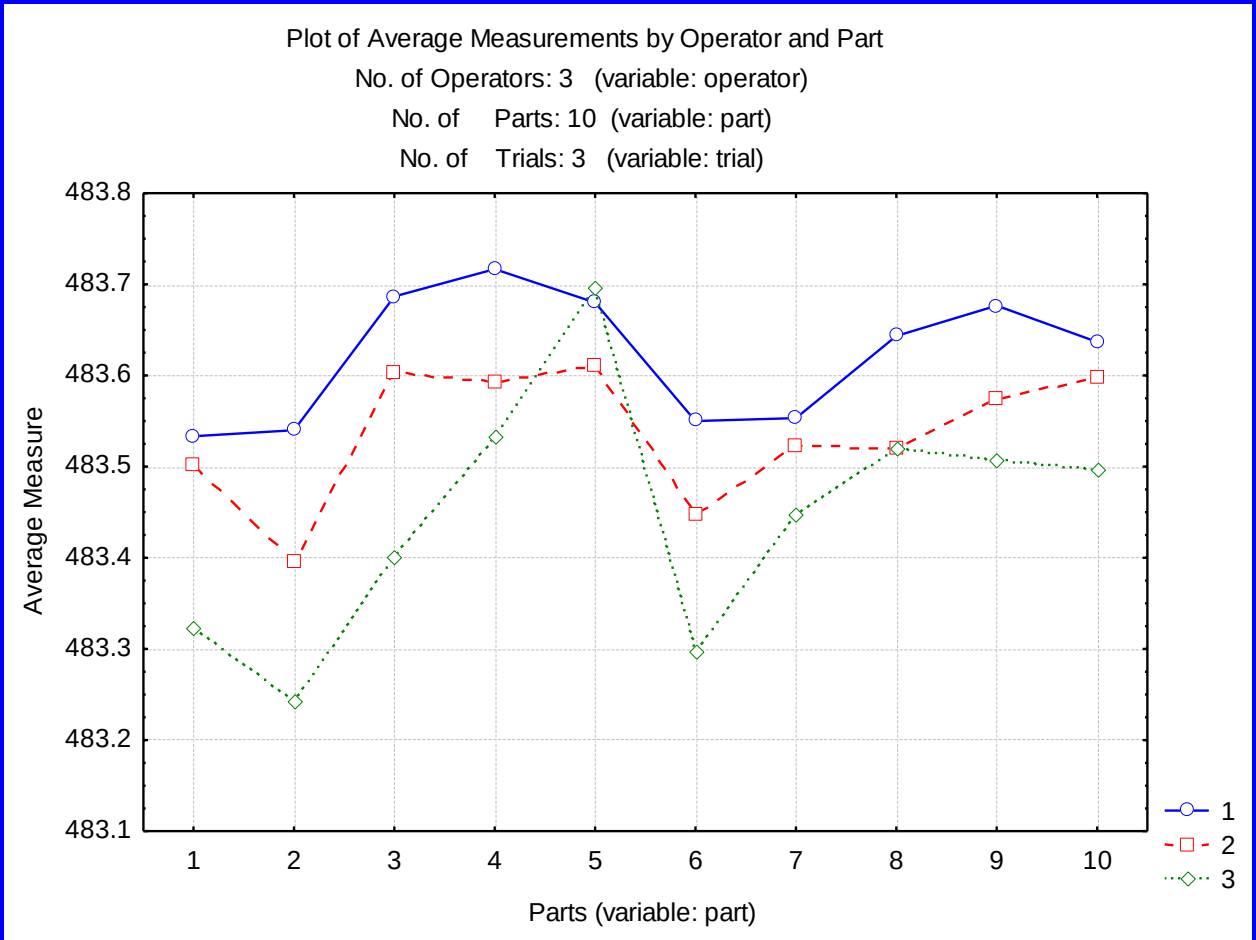
Perform a Gauge R&R study!

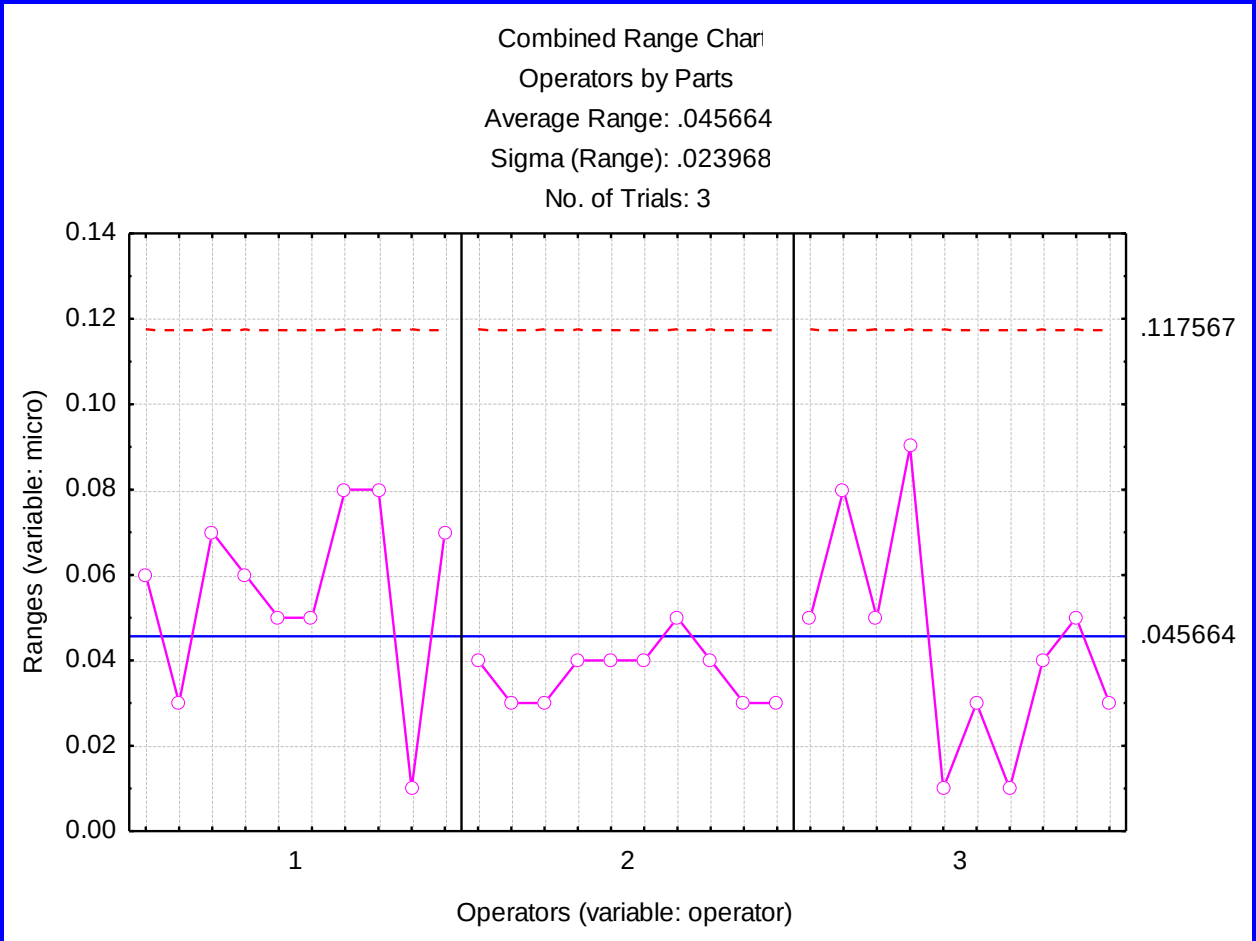
micro.sta

Statistics>Industrial Statistics & Six Sigma>Process Analysis>
>Gage repeatability and reproducibility


Gage Repeatability & Reproducibility Results: micro.st
Variable: micro Mean: 483.535 Std.Dev.
Operators (operator): 3 Parts (part): 10
Quick
Advanced
Descriptives/plots
Gage performance
Options
Options for percent tolerance analysis
Total tolerance for parts: 1.52
Number of sigma intervals: 6
Enter the tolerance value and the desired sigma intervals to compute gage capability as percent of total tolerance.

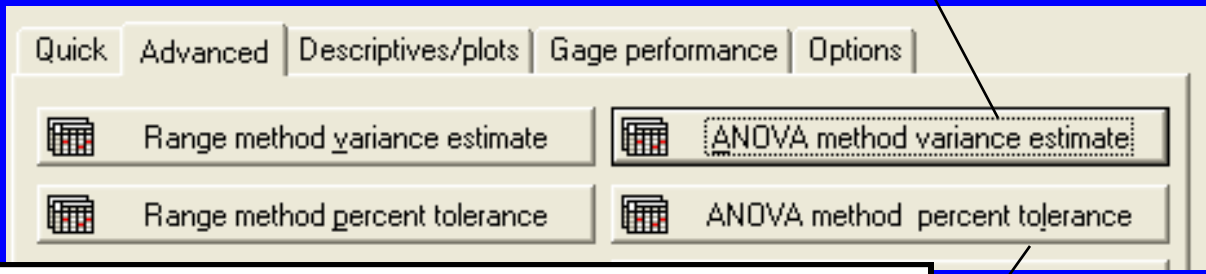

Repeatability & Reproducibility Summary Plo
No. of Operators: 3 (variable: operator)
No. of Parts: 10 (variable: part)
No. of Trials: 3 (variable: trial)
Deviation from Average
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
1
2
3
Operators (variable: operator)


Plot of Average Measurements by Operator and Part
No. of Operators: 3 (variable: operator)
No. of Parts: 10 (variable: part)
No. of Trials: 3 (variable: trial)
Average Measure
483.8
483.7
483.6
483.5
483.4
483.3
483.2
483.1
1
2
3
4
5
6
7
8
9
10
Parts (variable: part)
1
2
3


Combined Range Chart
Operators by Parts
Average Range: .045664
Sigma (Range): .023968
No. of Trials: 3
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Ranges (variable: micro)
.117567
.045664
1
2
3
Operators (variable: operator)

Variance Components; Variable: micro (micro.sta)
Mean=483.535 Std.Dv=.119260
Operators: 3 Parts: 10 Trials: 3

Source (Expected MS)	Estimatd Sigma	.90 Lowr Conf.Lim	.90 Uppr Conf.Lim	Estimatd Variance	% of R & R	% of Total
Repeatability	0.026055	0.022695	0.030711	0.000679	6.2345	3.9321
Operator	0.085921	0.043097	0.386196	0.007382	67.7951	42.7588
Interaction (OP)	0.053179	0.038615	0.077027	0.002828	25.9704	16.3797
Part-to-Part	0.079849	0.044634	0.144458	0.006376		36.9294
Combined R & R	0.104352	0.075945	0.391256	0.010889	100.0000	63.0706
Total	0.131397			0.017265		100.0000



Percent Tolerance Analysis:micro Sigma intervals:6. (micro.sta)
Mean=483.535 Std.Dv=.119260
Operators: 3 Parts: 10 Trials: 3

Source (Expected MS)	Measrmnt Units	.90 Lowr Conf.Lim	.90 Uppr Conf.Lim	% Proc. Variatn	% Total Contrib.	% of Tolernce
Repeatability (Equipment Var).	0.156333	0.136172	0.184266	19.8296	3.9321	10.2851
Operator (Appraiser Var.)	0.515525	0.258584	2.317176	65.3902	42.7588	33.9161
Interaction (Operator x Part)	0.319073	0.231688	0.462160	40.4718	16.3797	20.9916
Part Variation	0.479096	0.267801	0.866750	60.7695	36.9294	31.5195
Combined R & R	0.626110	0.455672	2.347534	79.4170	63.0706	41.1914
Total Process Variation	0.788382			100.0000	100.0000	51.8673
Tolerance	1.520000					100.0000

Example 16

Measure the height of 4 persons by 3 operators with 3 repetitions

operator	A			B			C		
part	rept 1	rept 2	rept 3	rept 1	rept 2	rept 3	rept 1	rept 2	rept 3
1									
2									
3									
4									

Relating the variance components to the total variance

source of fluctuation	$\hat{\sigma}$	$\hat{\sigma}^2$	% in R&R	% in total variation
(1) repeatability				
(2) operator				
(3) operator*part				
(2+3) reproducibility				
(1+2+3) R & R			100	
(4) parts				
(1+2+3+4) total				100.00

Comparisons for the 99% fluctuation range

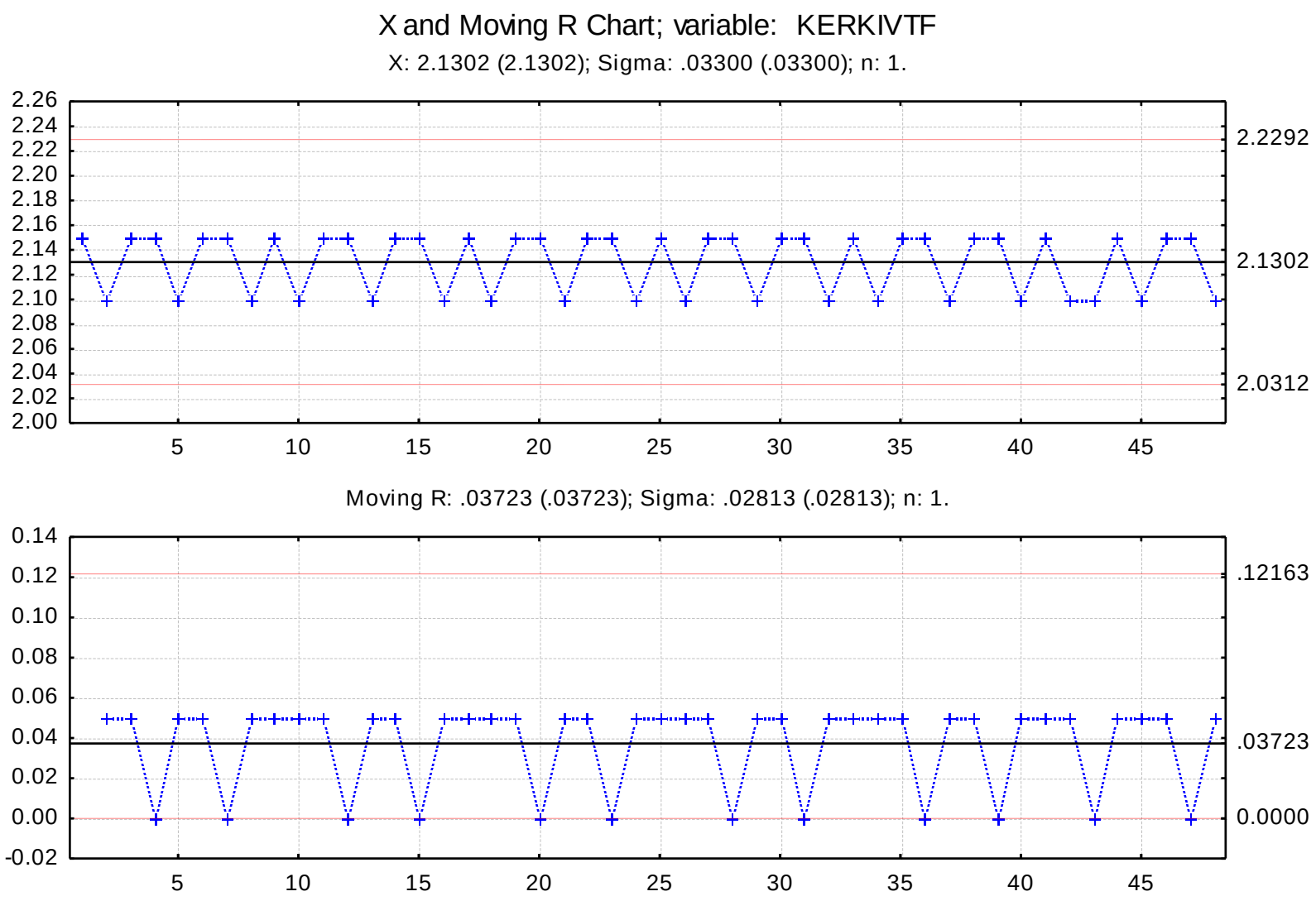
it is not from the summation in the first column!

source of fluctuation	$\hat{\sigma}$	99% range of fluctuation ($5.15 \cdot \hat{\sigma}$)	% in the total fluctuation	% in the spec. range
repeatability (gauge)				
reproducibility (operator)				
R & R				
parts				
total fluctuation			100.00	
spec. range				100.00

Number of distinguishable categories:

$$\frac{\hat{\sigma}_{\text{part}}}{\hat{\sigma}_{\text{R\&R}}}\sqrt{2} =$$

Improper measurement system:



Attribute gage R&R

The simplest method of study

E.g. 20 „parts” are taken in a random way, all of them is checked by at least two operators, at least twice

operator 1		operator 2		reference	decision on the study
rept 1	rept 2	rept 1	rept 2		
good	good	good	good	good	good result
bad	bad	bad	bad	bad	good result
good	good	good	good	bad	bias
good	good	bad	bad		reproducibility!
good	bad	good	good		repeatability!

Example 26

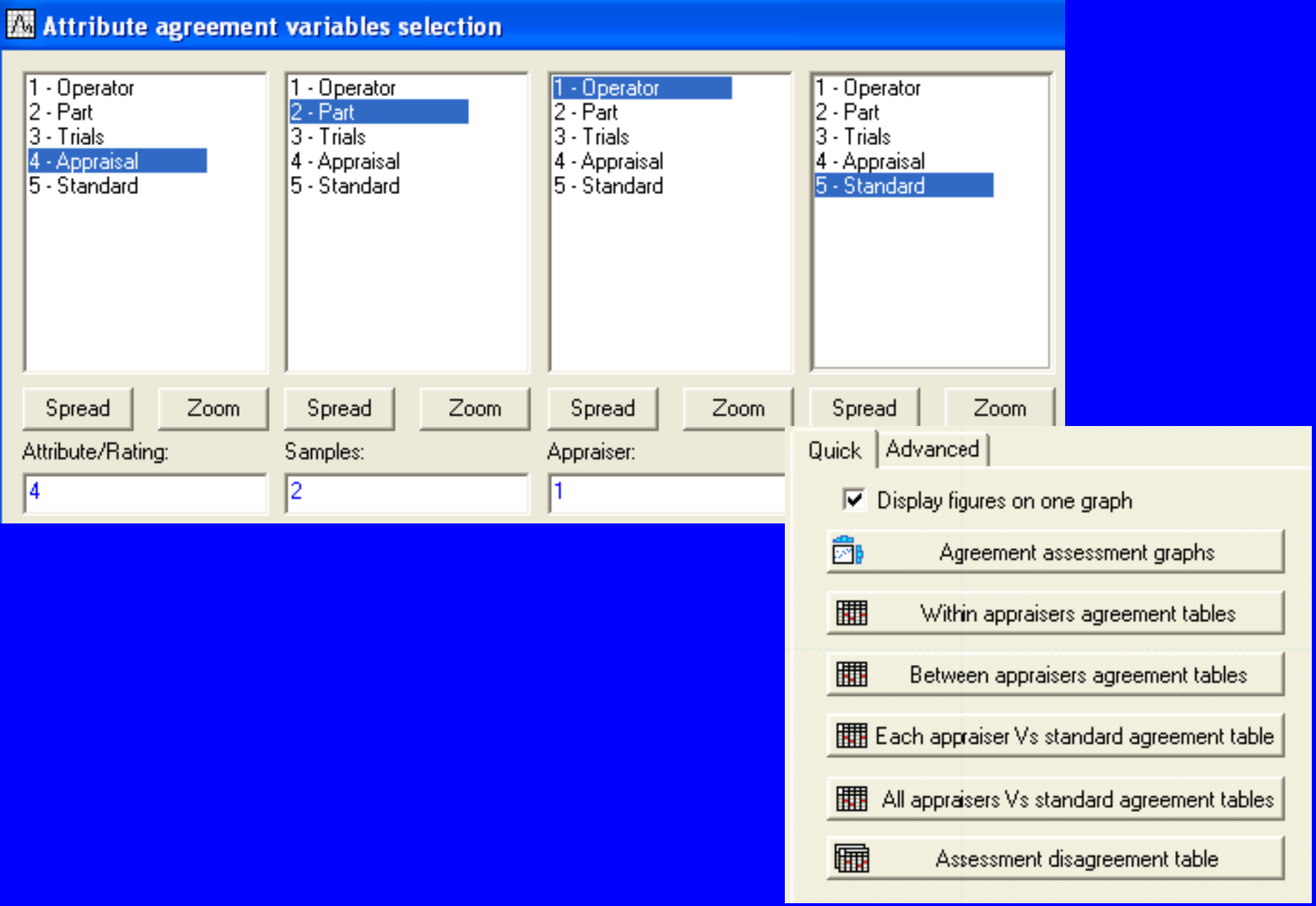
(Statistica example)

7 newly hired operators are asked to judge all of 5 parts for 3 trials to determine whether a product is considered good or bad, Go (Accept) or No-Go (Reject).

MSA_Attribute.sta (File>Open Examples)

	Operator	Part	Trials	Appraisal	Standard
1	Karthic	1	1	Go	Go
2	Karthic	2	1	No-Go	No-Go
3	Karthic	3	1	Go	No-Go
4	Karthic	4	1	Go	Go
5	Karthic	5	1	Go	Go
6	Karthic	1	2	Go	Go
7	Karthic	2	2	No-Go	No-Go
8	Karthic	3	2	No-Go	No-Go
9	Karthic	4	2	Go	Go
10	Karthic	5	2	Go	Go
11	Karthic	1	3	Go	Go
12	Karthic	2	3	No-Go	No-Go
13	Karthic	3	3	No-Go	No-Go
14	Karthic	4	3	Go	Go
15	Karthic	5	3	Go	Go
16	Siva	1	1	Go	Go
17	Siva	2	1	Go	No-Go
18	Siva	3	1	No-Go	No-Go

Statistics>Industrial Statistics & Six Sigma>Process Analysis>
>Attribute agreement analysis


Attribute agreement variables selection
1 - Operator
2 - Part
3 - Trials
4 - Appraisal
5 - Standard
Spread
Zoom
Attribute/Rating:
4
Samples:
2
Appraiser:
1
Quick
Advanced
Display figures on one graph
Agreement assessment graphs
Within appraisers agreement tables
Between appraisers agreement tables
Each appraiser Vs standard agreement table
All appraisers Vs standard agreement tables
Assessment disagreement table

Within Appraisers (MSA_Attribute.sta)
Attribute Agreement Analysis for Appraisal

Appraiser	# Inspected	# Matched	Percent	95% Lower CI	95% Upper CI
Jennifer	5	2	40.00	5.27	85.34
Karthic	5	4	80.00	28.36	99.49
Kiron	5	1	20.00	0.51	71.64
Krista	5	4	80.00	28.36	99.49
Nitin	5	4	80.00	28.36	99.49
Sachin	5	5	100.00	54.93	100.00
Siva	5	4	80.00	28.36	99.49

Between Appraisers (MSA_Attribute.sta)
Attribute Agreement Analysis for Appraisal

Assessment Agreement				
# Inspected	# Matched	Percent	95% Lower CI	95% Upper CI
5	0.	0.00	0.00	45.07

Each Appraiser vs Standard (MSA_Attribute.sta)
Attribute Agreement Analysis for Appraisal

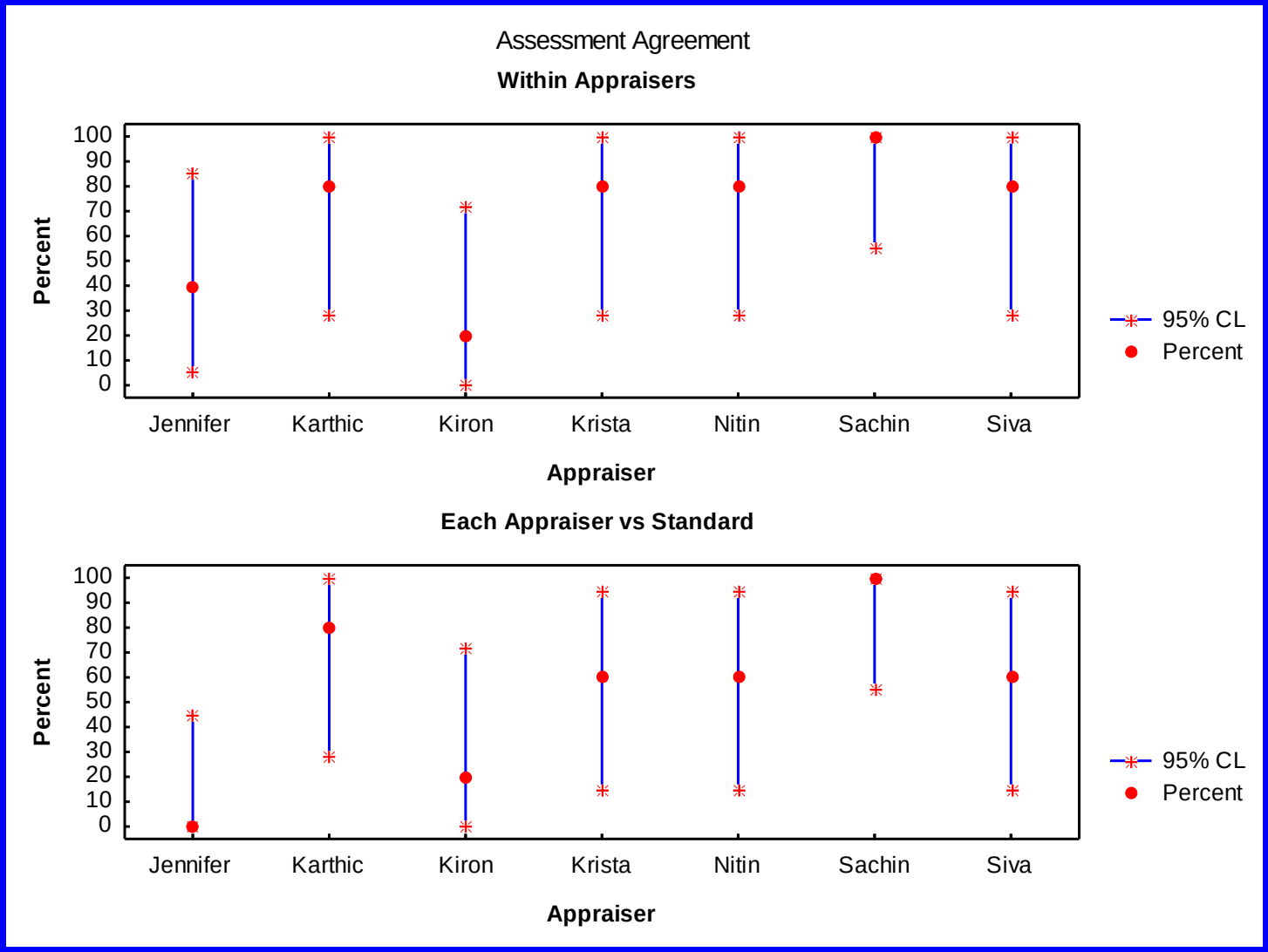
Appraiser	# Inspected	# Matched	Percent	95% Lower CI	95% Upper CI
Jennifer	5	0.	0.00	0.00	45.07
Karthic	5	4	80.00	28.36	99.49
Kiron	5	1	20.00	0.51	71.64
Krista	5	3	60.00	14.66	94.73
Nitin	5	3	60.00	14.66	94.73
Sachin	5	5	100.00	54.93	100.00
Siva	5	3	60.00	14.66	94.73

All Appraisers vs Standard (MSA_Attribute.sta)
Attribute Agreement Analysis for Appraisal

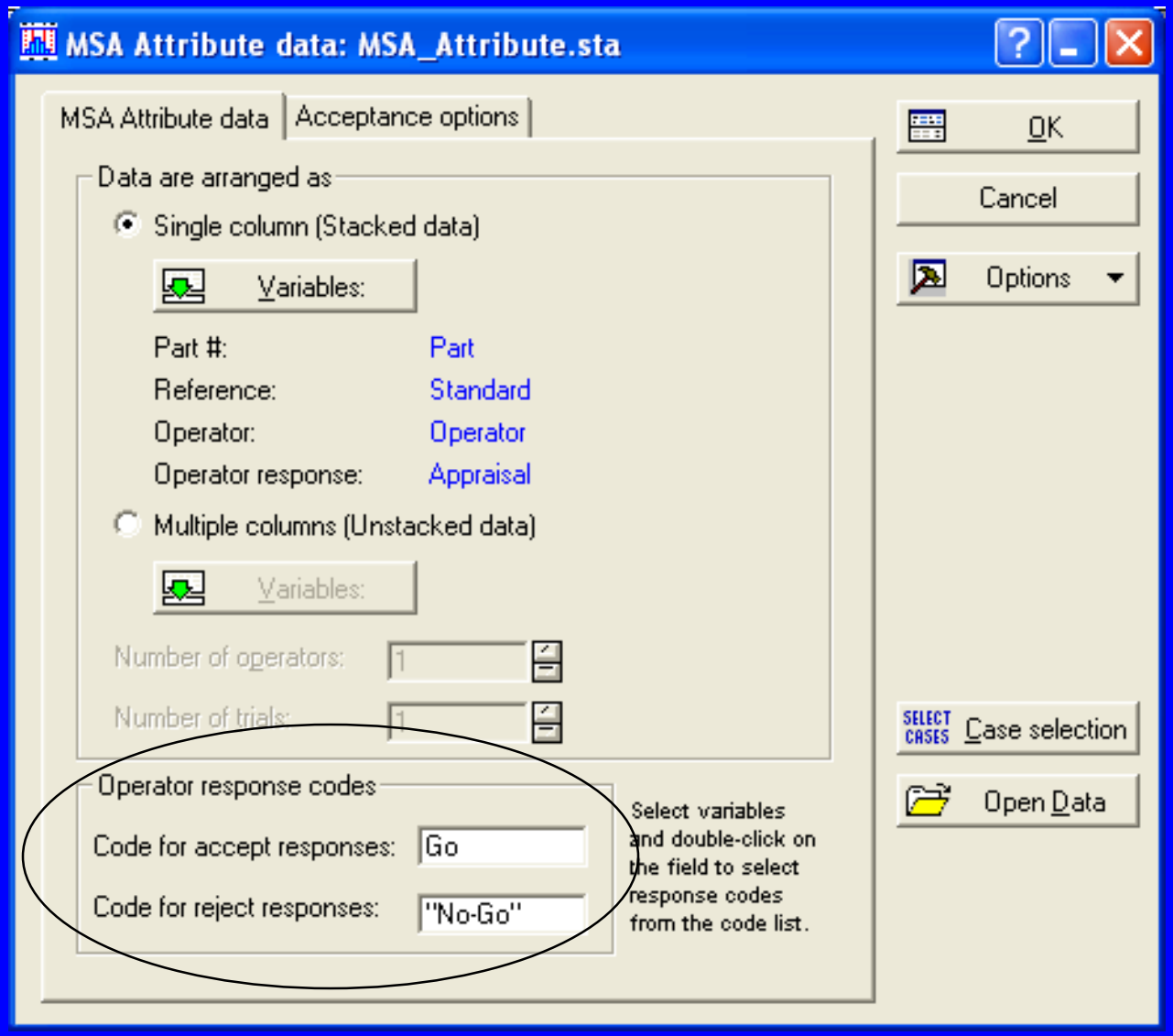
# Inspected	# Matched	Percent	95% Lower CI	95% Upper CI
5	0.	0.00	0.00	45.07

Assessment Disagreement (MSA_Attribute.sta)
Go / No-Go: Assessments across trials = Go / standard = No-Go
No-Go / Go: Assessments across trials = No-Go / standard = Go

Appraiser	# Go / No-Go	Percent	# No-Go / Go	Percent	# Mixed	Percent
Jennifer	0	0.00	2	66.67	3	60.00
Karthic	0	0.00	0	0.00	1	20.00
Kiron	0	0.00	0	0.00	4	80.00
Krista	1	50.00	0	0.00	1	20.00
Nitin	0	0.00	1	33.33	1	20.00
Sachin	0	0.00	0	0.00	0	0.00
Siva	1	50.00	0	0.00	1	20.00


Assessment Agreement
Within Appraisers
Percent
100
90
80
70
60
50
40
30
20
10
0
Jennifer
Karthic
Kiron
Krista
Nitin
Sachin
Siva
Appraiser
95% CL
Percent
Each Appraiser vs Standard
Percent
100
90
80
70
60
50
40
30
20
10
0
Jennifer
Karthic
Kiron
Krista
Nitin
Sachin
Siva
Appraiser
95% CL
Percent

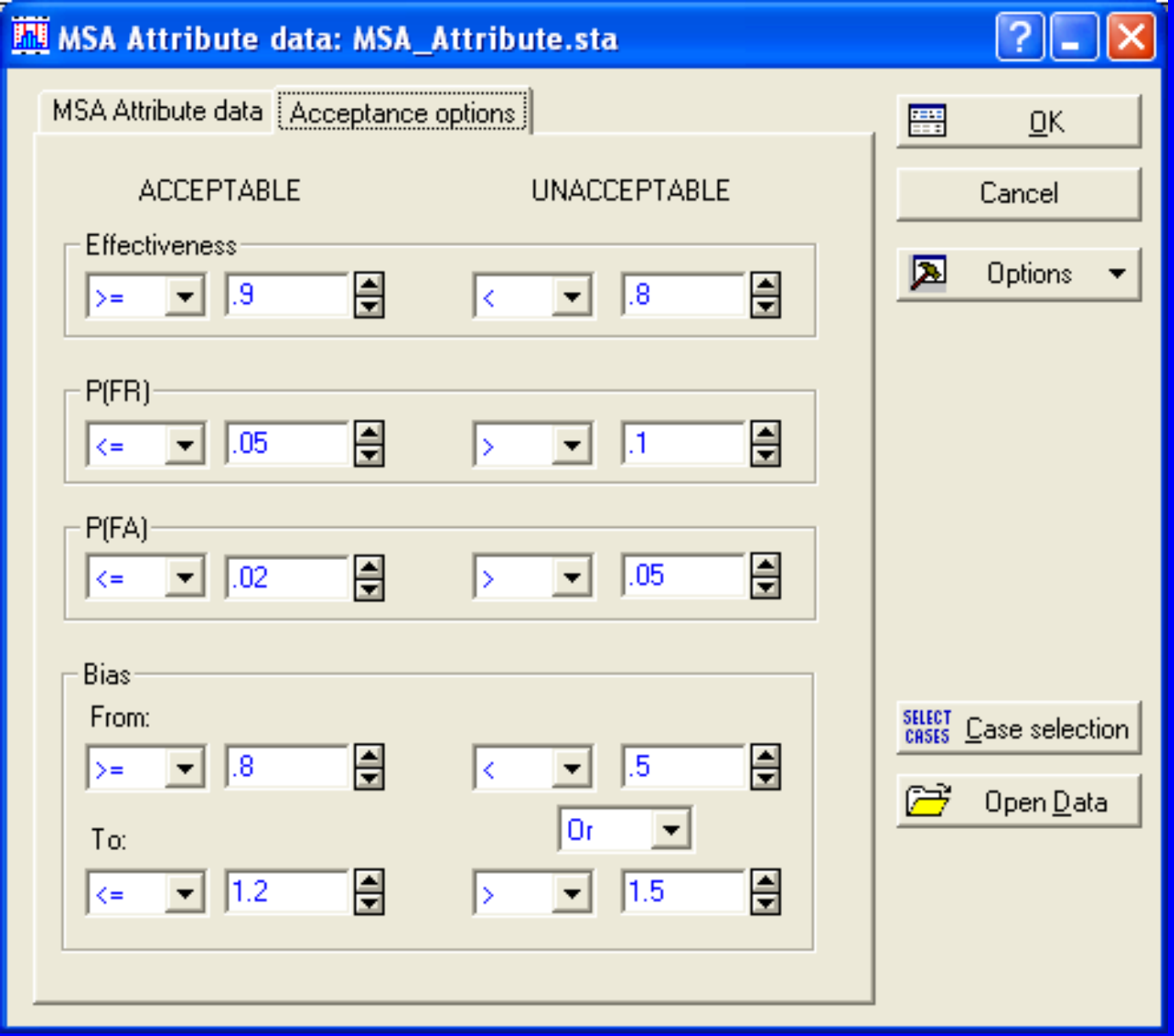
Statistics>Industrial Statistics & Six Sigma>Process Analysis>>MSA Attribute Data



	Attribute MSA Analysis (MSA_Attribute.sta) Number and Type Mistake By Operator							
	Truth	Jennifer	Karthic	Kiron	Krista	Nitin	Sachin	Siva	Total
False rejects	Go	7	0	3	0	3	0	1	14
False accepts	No-Go	2	1	2	4	1	0	3	13
Number of correct decisions		6	14	10	11	11	15	11	78
Total opportunities for a decision		15	15	15	15	15	15	15	105
Total opportunities to rate good parts		9	9	9	9	9	9	9	63
Total opportunities to rate bad parts		6	6	6	6	6	6	6	42

	Attribute MSA Analysis (MSA_Attribute.sta) Inspection Capability						
	Jennifer	Karthic	Kiron	Krista	Nitin	Sachin	Siva
Effectiveness	Unacceptable	Acceptable	Unacceptable	Unacceptable	Unacceptable	Acceptable	Unacceptable
P(FR)	Unacceptable	Acceptable	Unacceptable	Acceptable	Unacceptable	Acceptable	Unacceptable
P(FA)	Unacceptable	Unacceptable	Unacceptable	Unacceptable	Unacceptable	Acceptable	Unacceptable
Bias	Unacceptable	Unacceptable	Acceptable	Unacceptable	Unacceptable		Unacceptable

	Attribute MSA Analysis (MSA_Attribute.sta) Inspection Capability							
	Jennifer	Karthic	Kiron	Krista	Nitin	Sachin	Siva	Total
Effectiveness	0.40000	0.93333	0.66667	0.73333	0.73333	1.00000	0.73333	0.74286
P(FR)	0.77778	0.00000	0.33333	0.00000	0.33333	0.00000	0.11111	0.22222
P(FA)	0.33333	0.16667	0.33333	0.66667	0.16667	0.00000	0.50000	0.30952
Bias	2.33333	0.00000	1.00000	0.00000	2.00000		0.22222	0.71795


MSA Attribute data: MSA_Attribute.sta
MSA Attribute data
Acceptance options
OK
Cancel
Options
ACCEPTABLE
UNACCEPTABLE
Effectiveness
>=
.9
<
.8
P(FR)
<=
.05
>
.1
P(FA)
<=
.02
>
.05
Bias
From:
>=
.8
<
.5
Or
To:
<=
1.2
>
1.5
SELECT CASES
Case selection
Open Data

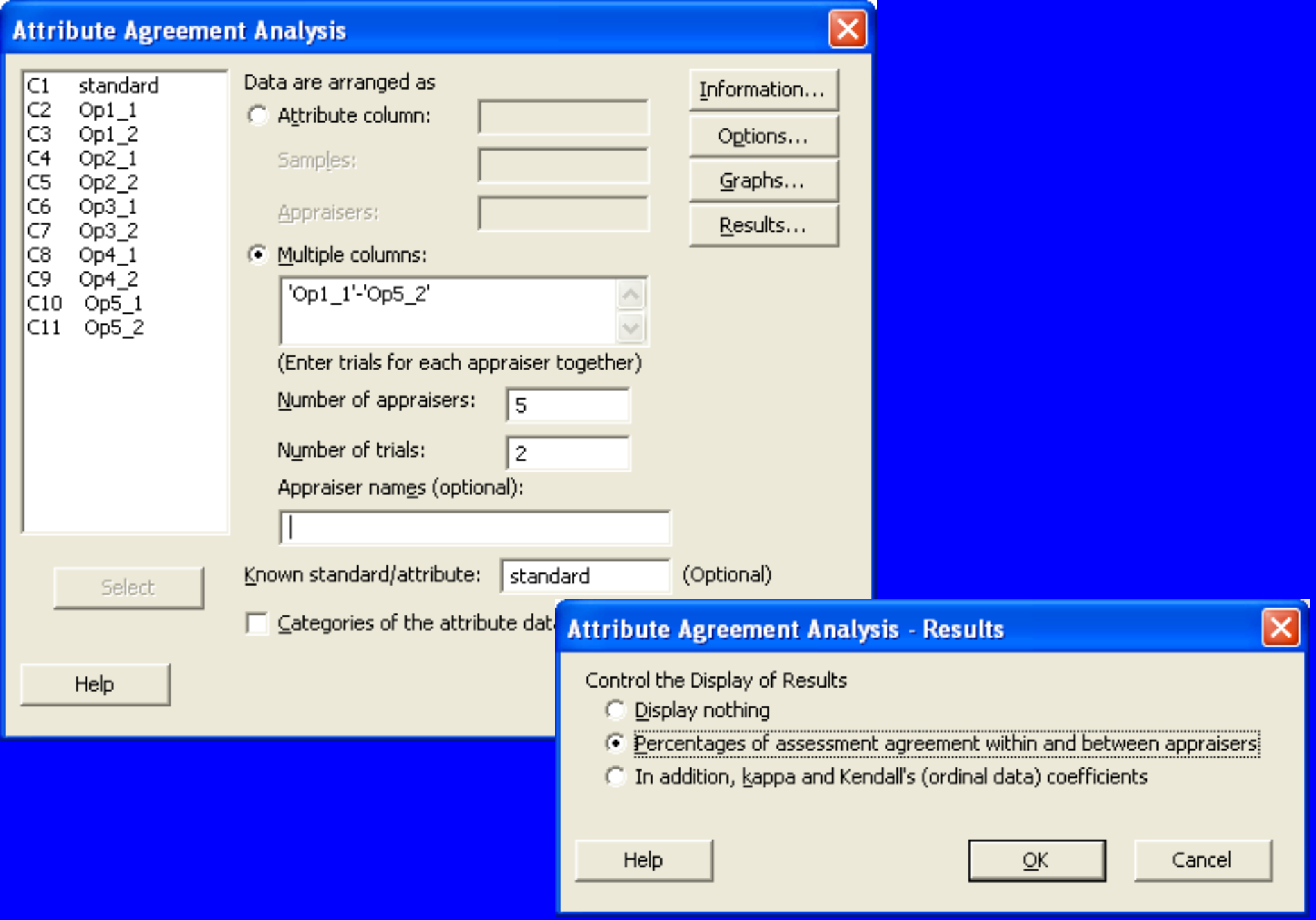
Example 55

5 operators assess twice the quality of a product

attrib.mtw

standard	Op1_1	Op1_2	Op2_1	Op2_2	Op3_1	Op3_2
1	1	1	1	1	1	1
1	0	0	0	0	1	1
1	1	1	1	1	1	1
0	0	0	0	0	0	0
1	1	1	1	1	1	1
0	0	0	0	0	0	0
1	1	1	1	1	1	1

Stat>Quality Tools>Attribute Agreement Analysis


Attribute Agreement Analysis
C1 standard
C2 Op1_1
C3 Op1_2
C4 Op2_1
C5 Op2_2
C6 Op3_1
C7 Op3_2
C8 Op4_1
C9 Op4_2
C10 Op5_1
C11 Op5_2
Data are arranged as
Attribute column:
Samples:
Appraisers:
Multiple columns:
'Op1_1'-'Op5_2'
(Enter trials for each appraiser together)
Number of appraisers: 5
Number of trials: 2
Appraiser names (optional):
Known standard/attribute: standard
(Optional)
Categories of the attribute dat
Information...
Options...
Graphs...
Results...
Select
Help
Attribute Agreement Analysis - Results
Control the Display of Results
Display nothing
Percentages of assessment agreement within and between appraisers
In addition, kappa and Kendall's (ordinal data) coefficients
Help
OK
Cancel

Attribute Agreement Analysis

Attribute Agreement Analysis for Op1_1, Op1_2, Op2_1, Op2_2, Op3_1, Op3_2, ...

Within Appraisers

Assessment Agreement

Appraiser	# Inspected	# Matched	Percent	95 % CI
1	20	20	100.00	(86.09, 100.00)
2	20	19	95.00	(75.13, 99.87)
3	20	20	100.00	(86.09, 100.00)
4	20	19	95.00	(75.13, 99.87)
5	20	18	90.00	(68.30, 98.77)

Matched: Appraiser agrees with him/herself across trials.

Each Appraiser vs Standard

Assessment Agreement

Appraiser	# Inspected	# Matched	Percent	95 % CI
1	20	19	95.00	(75.13, 99.87)
2	20	18	90.00	(68.30, 98.77)
3	20	19	95.00	(75.13, 99.87)
4	20	18	90.00	(68.30, 98.77)
5	20	16	80.00	(56.34, 94.27)

Matched: Appraiser's assessment across trials agrees with the known standard.

Assessment Disagreement

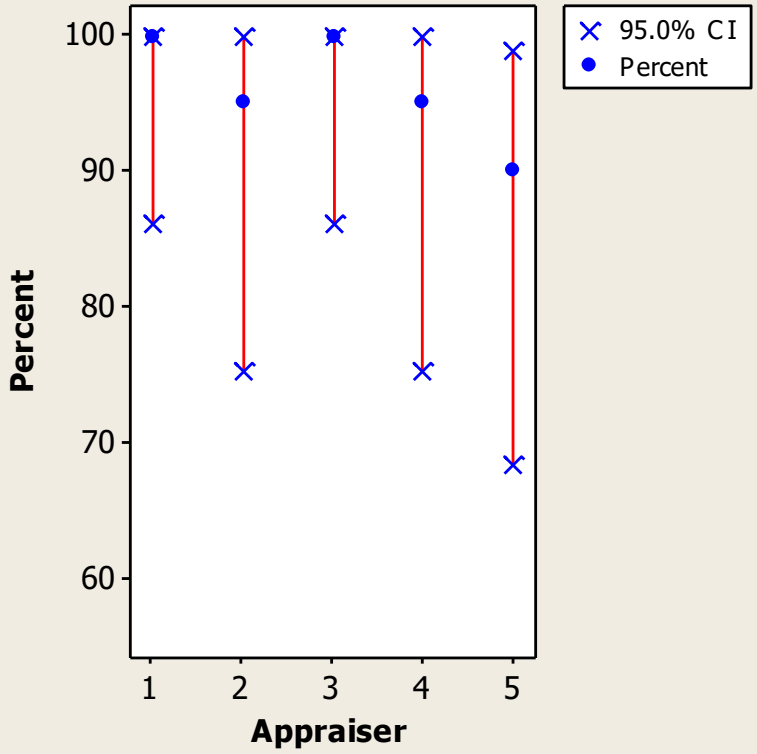
Appraiser	# 1 / 0	Percent	# 0 / 1	Percent	# Mixed	Percent
1	0	0.00	1	10.00	0	0.00
2	0	0.00	1	10.00	1	5.00
3	1	10.00	0	0.00	0	0.00
4	0	0.00	1	10.00	1	5.00
5	1	10.00	1	10.00	2	10.00

1 / 0: Assessments across trials = 1 / standard = 0.
0 / 1: Assessments across trials = 0 / standard = 1.
Mixed: Assessments across trials are not identical.

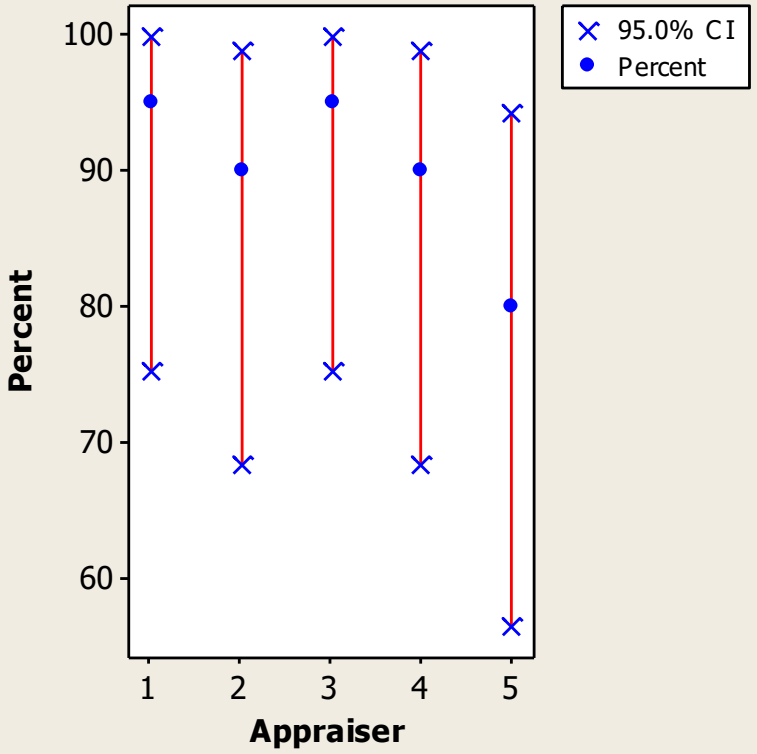
Assessment Agreement

Date of study:
Reported by:
Name of product:
Misc:

Within Appraisers



Appraiser vs Standard



Between Appraisers

Assessment Agreement

# Inspected	# Matched	Percent	95 % CI
20	15	75.00	(50.90, 91.34)

Matched: All appraisers' assessments agree with each other.

All Appraisers vs Standard

Assessment Agreement

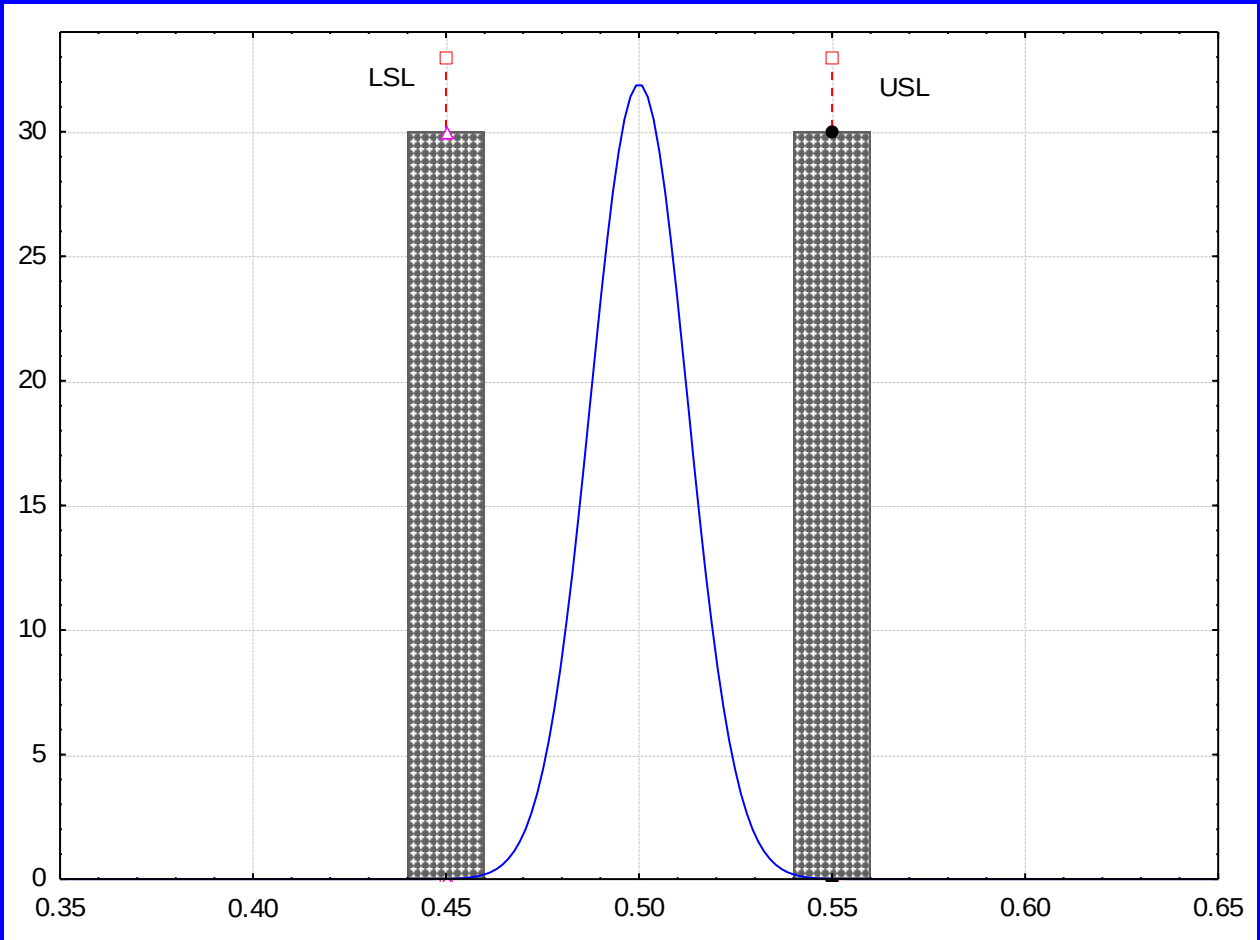
# Inspected	# Matched	Percent	95 % CI
20	15	75.00	(50.90, 91.34)

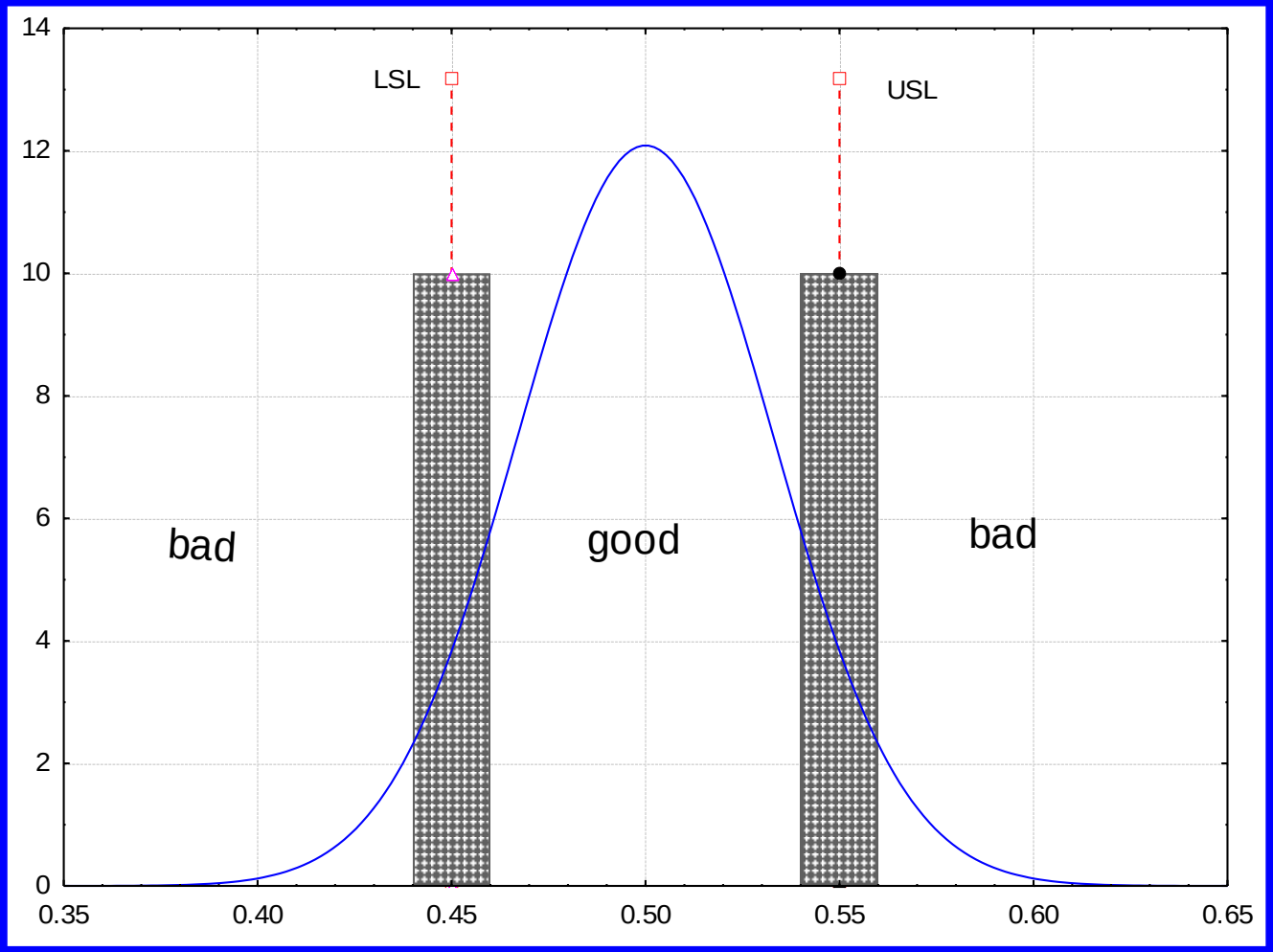
Matched: All appraisers' assessments agree with the known standard.

Example 27

All participants assess twice if print on M&M candies is OK or not

depends on data (process)




LSL
USL
bad
good
bad
14
12
10
8
6
4
2
0
0.35
0.40
0.45
0.50
0.55
0.60
0.65