residual mean square

$$s_r^2 = \frac{\sum \left(y_i - \hat{Y}_i\right)^2}{n-2} = \frac{\chi^2 \sigma_y^2}{\nu} = s_y^2$$

It is an estimate of the variance of experimental errors:

$$\hat{\sigma}_y^2 = s_y^2 = s_r^2$$

Regression Summary for Dependent Variable: y (regr1)
R= .95061604 R²= .90367086 Adjusted R²= .87958858
F(1,4)=37.524 p<.00360 Std.Error of estimate: .62136

N=6	Beta	Std.Err. of Beta	B	Std.Err. of B	t(4)	p-level
Intercept			0.05196	0.504033	0.103084	0.922858
x	0.950616	0.155185	32.01651	5.226581	6.125708	0.003598

Properties of the estimators:

$$\hat{Y}_i = a + b\left(x_i - \bar{x}\right)$$

$$Y_i = \alpha + \beta\left(x_i - \bar{x}\right)$$

$$E(a) \equiv E\left(\frac{\sum y_i}{n}\right) = \alpha$$

$$Var(a) = \frac{\sum \sigma_y^2}{(n)^2} = \frac{\sigma_y^2}{n}$$

$$E(b) = \beta$$

$$Var(b) = \frac{\sum \left(x_i - \bar{x}\right)^2 \sigma_y^2}{\left(\sum \left(x_i - \bar{x}\right)^2\right)^2} = \frac{\sigma_y^2}{\sum \left(x_i - \bar{x}\right)^2}$$

$$E(\hat{Y}) = E[a + b(x - \bar{x})] = \alpha + \beta(x - \bar{x}) = Y$$

$$Var(\hat{Y}) = Var(a) + (x - \bar{x})^2 Var(b) = \sigma_y^2 \left[\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2} \right]$$

How could this be decreased?

n, σ^2, x (allocation)

Estimates of the variances – Standard deviations

s_r^2 is substituted for σ_y^2

$$Var(a) = \frac{\sigma_y^2}{n} \qquad s_a = \frac{s_y}{\sqrt{n}}$$

$$Var(b) = \frac{\sigma_y^2}{\sum (x_i - \bar{x})^2} \qquad s_b = \frac{s_y}{\sqrt{\sum (x_i - \bar{x})^2}}$$

$$\hat{\sigma}_y^2 = s_y^2 = s_r^2$$

Confidence band: Where does the true line lie?

$$t = \frac{\hat{Y} - Y}{s_{\hat{Y}}}$$

$$\hat{Y} - t_{\alpha/2} s_{\hat{Y}} < Y < \hat{Y} + t_{\alpha/2} s_{\hat{Y}}$$

$$s_{\hat{Y}} = s_y \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$$

Can the measurements fall out of this band?

Prediction band: Where can a new measurement fall?

$$\hat{Y} - t_{\alpha/2} s_{y^* - \hat{Y}} < y^* < \hat{Y} + t_{\alpha/2} s_{y^* - \hat{Y}}$$

$$s_{y^* - \hat{Y}} = s_r \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} = \sqrt{s_r^2 + s_a^2 + s_b^2 (x - \bar{x})^2}$$

Comparison of the confidence and prediction bands

blue – confidence band
red – prediction band

$$s_{\hat{Y}} = s_y \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$$

$$s_{y^*-\hat{Y}} = s_r \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$$