Problem 1

Data points:

x	y
0.6	5.16
1.4	7.33
2.5	9.24
3.7	11.05
4.8	13.34
5.6	15.66

questions:

1. Calculate the residual mean square. (0.1688)
2. Construct a 90% confidence interval for the slope.
3. Test whether the intercept is 0. (At $\alpha = 5\%$ level.)

43. Test whether the intercept is 4. (At $\alpha = 5\%$ level.) (t0=0.504, p=0.641)

5. In what interval can you find the point of the true regression line at x =3 with 99% probability? (9.33, 10.87)
6. In what interval can you find a new measurement at x = 2 with 90% probability? (7.15, 9.09)

Effect	Param.	Std.Err	t	p	-95,00% Cnf.Lmt	+95,00% Cnf.Lmt
Intercept	4,170	0,337	12,38	0,00025	3,23	5,11
x	1,976	0,094	20,97	0,00003	1,71	2,24

Problem 2

Data points:

x	y
1.3	13.7
2.6	10.5
3.3	9.9
4.9	7.4
5.5	5.1

questions:

1. Calculate the residual mean square. (0.3582)
2. Construct a 99% confidence interval for the intercept. (12.02, 19.88)
3. Can we say that the slope is -2? (At $\alpha = 5\%$ level.)

(H0: beta = -2, t0 = 0.67, p = 0.551)

4. In what interval can you find the point of the true regression line at x = 5 with 95% probability? (5.337, 7.716)
5. In what interval can you find a new measurement at x = 2? (At $\alpha = 5\%$ level.) (9.94, 14.43)

Effect	Param.	Std.Err	t	p	-95,00% Cnf.Lmt	+95,00% Cnf.Lmt
Intercept	15,95	0,67	23,71	0,0002	13,81	18,09
x2	-1,88	0,18	-10,74	0,0017	-2,44	-1,33

Problem 3

8 jams were made with different cooking parameters. The quality of the jams was measured on a 0 to 100 point scale. The table below contains the cooking parameters and the quality of the result.

temperature		120 °C		180 °C	
cooking time		1 hrs	2 hrs	1 hrs	2 hrs
sugar	20 g	35	28	45	42
	50 g	44	39	48	44

Questions

1. Calculate all the effects.
2. Draw a main effect plot for cooking time. What can you deduct?
3. Draw an interaction plot for sugar/cooking time. What can you deduct?
4. Draw an interaction plot for cooking time/temperature. What can you deduct?
5. Draw a Pareto chart. Then based on it reduce the model.
6. Using the reduced model, make an estimate for the measurement at 2 hrs cooking time, 25 g sugar and 160 °C temperature.

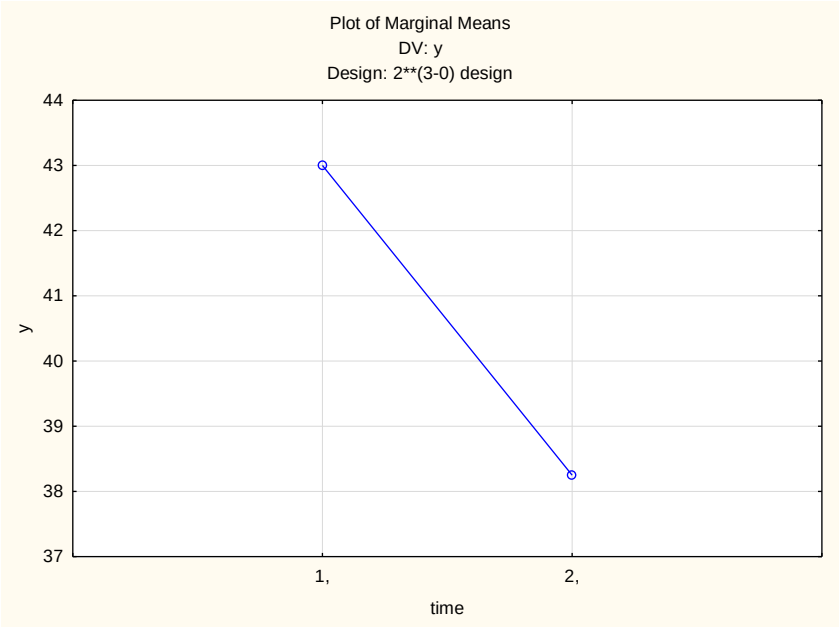
Solution

sugar	temp	time	y
20	120	1	35
20	120	2	28
20	180	1	45
20	180	2	42
50	120	1	44
50	120	2	39
50	180	1	48
50	180	2	44

1. Calculate all the effects.

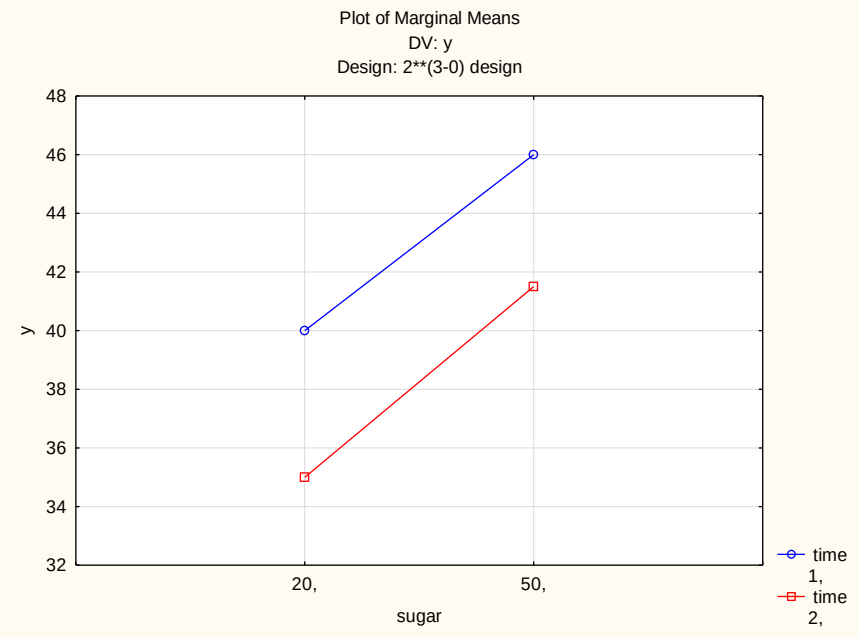
Factor	Effect
(1)sugar	6,25000
(2)temp	8,25000
(3)time	-4,75000

2. Draw a main effect plot for cooking time. What can you deduct?



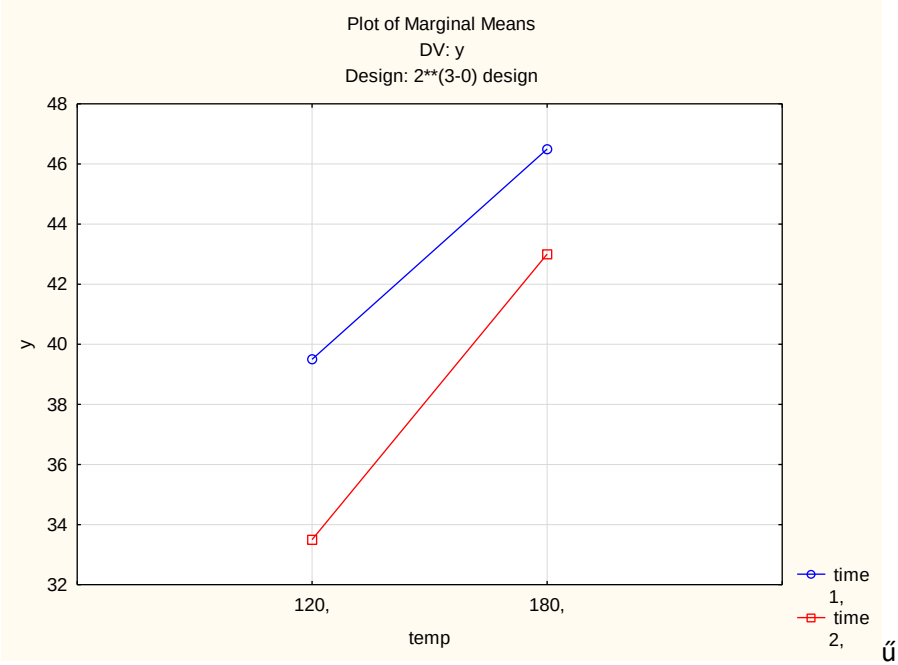
With longer cooking time the jam becomes less good. If the cooking time is increased from 1 hour to 2 hours the quality drops with 4.75 points.

3. Draw an interaction plot for sugar/cooking time. What can you deduct?



There is no interaction between the the amount of sugar and the cooking time. No matter how long is the cooking time (1 or 2 hours) adding more sugar (50 instead of 20) would improve the quality of the jam with the same amount.

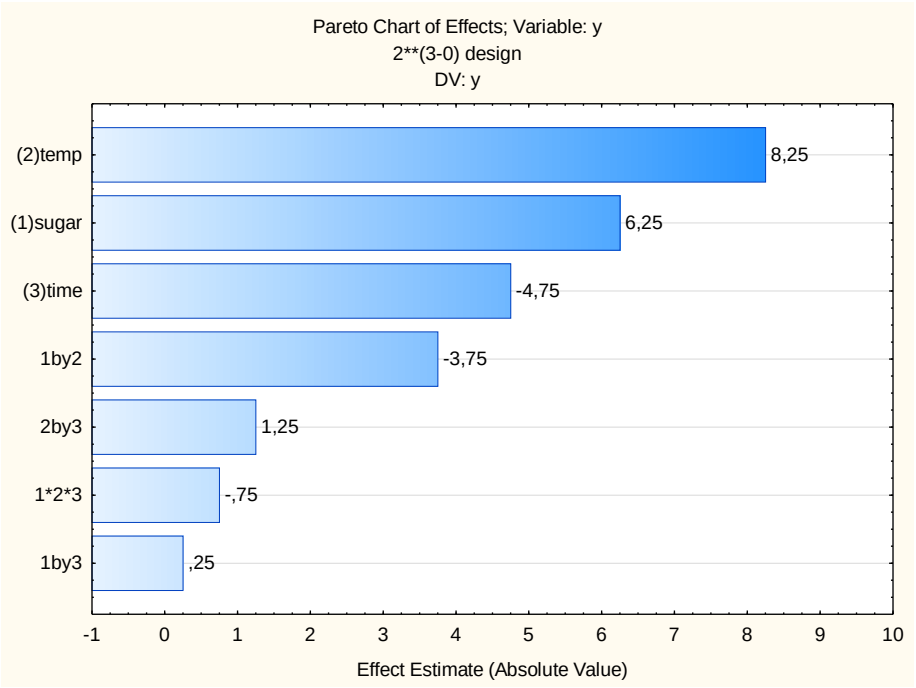
4. Draw an interaction plot for cooking time/temperature. What can you deduct?



ű

There is no interaction between the the temperature and the cooking time.

5. Draw a Pareto chart. Then based on it reduce the model.



The reduced model:

$\hat{Y} = 40.625 + 3.125x_1 + 4.125x_2 - 2.375x_3 - 1.875x_1x_2$

6. Using the reduced model, make an estimate for the measurement at 2 hrs cooking time, 25 g sugar and 160 °C temperature.

$\hat{Y} = 40.625 + 3.125 * 0 + 4.125 * 0 - 2.375 * 1 - 1.875 * 0 * 1 = 38.25$

Problem 4

The table below contains the design and the results of a set of expetiments. (The conversion is the outcome.)

pressure (bar)	temperature (°C)	pH	conversion (%)
0.5	20	5	2.76
1	20	5	26.79
0.5	30	5	38.90
1	30	5	31.37
0.5	20	7	3.08
1	20	7	26.39
0.5	30	7	39.10
1	30	7	30.38

Questions:

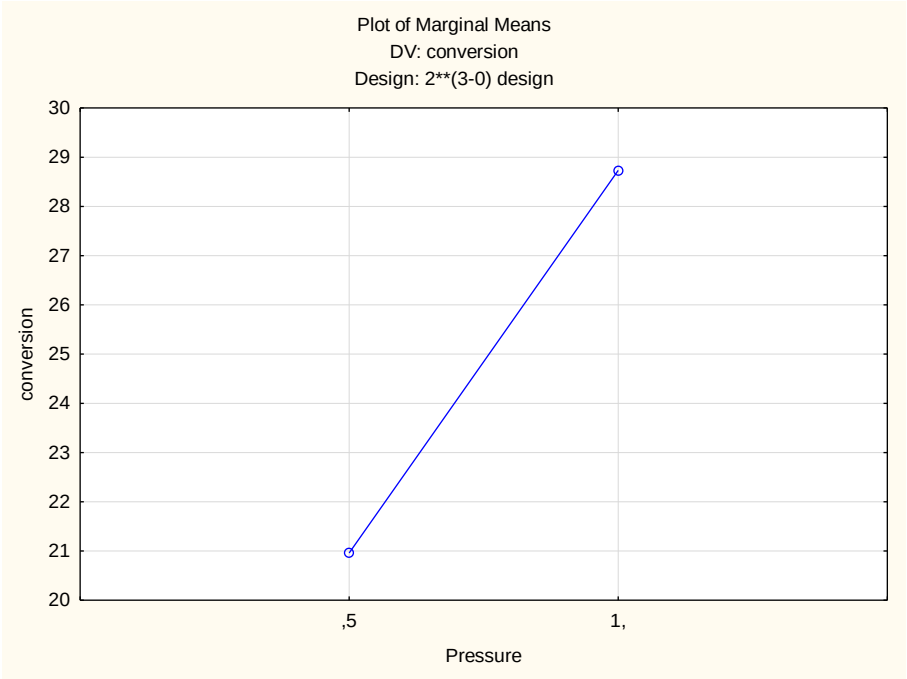
1. Calculate all the effects and coefficients.
2. Draw a main effect plot for the pressure. What can you deduct?
3. Draw an interaction plot for the temperature/pressure. What can you deduct?
4. Draw a Pareto chart. Then based on it reduce the model.
5. Using the reduced model, make an estimate for the conversion at 0.6 bar, 29 °C and 5.5 pH.

Solution

1. Calculate all the effects and coefficients.

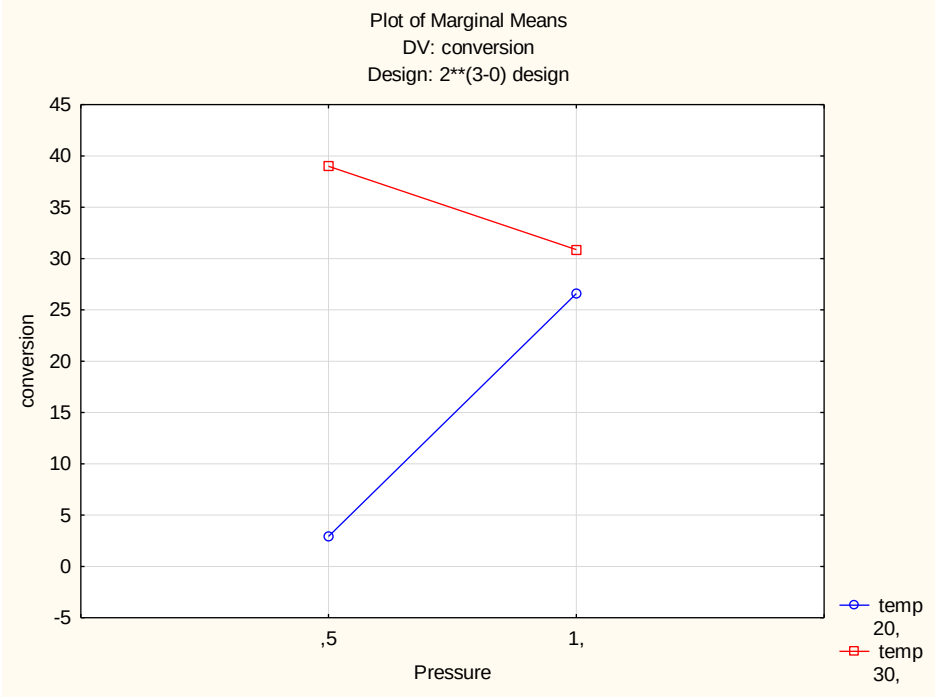
Factor	Effect	Coefficient
Intercept		24,85
(1)Pressure	7,8	3,89
(2)temp	20,2	10,09
(3)pH	-0,2	-0,11
1 by 2		-7,95
1 by 3		-0,24
2 by 3		-0,09
1*2*3		-0,06

2. Draw a main effect plot for the pressure. What can you deduct?



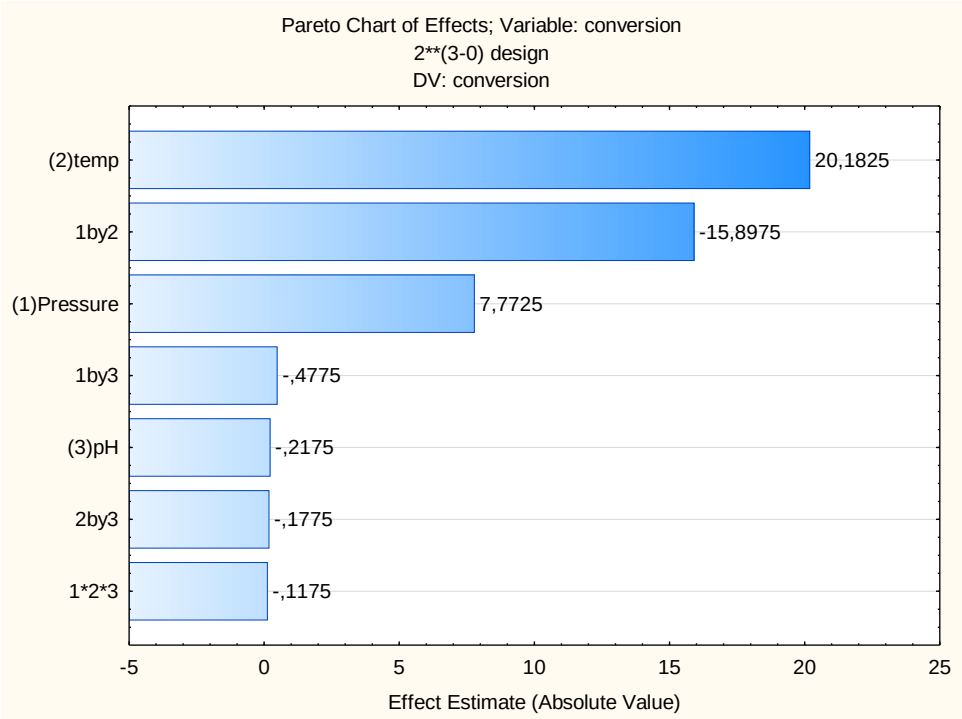
The conversion is higher at higher pressure. If the pressure is increased from 0.5 bar to 1 bar the conversion increases with 3.89%.

3. Draw an interaction plot for the temperature/pressure. What can you deduct?



There is interaction between the temperature and the pressure.

4. Draw a Pareto chart. Then based on it reduce the model.



The reduced model:

$\hat{Y} = 24.8 + 3.9x_1 + 10.1x_2 - 7.9x_1x_2$

5. Using the reduced model, make an estimate for the conversion at 0.6 bar, 29 °C and 5.5 pH.

$$\hat{Y} = 24.8 + 3.9 * \frac{0.6 - 0.75}{0.25} + 10.1 * \frac{29 - 25}{5} - 7.9 * \frac{0.6 - 0.75}{0.25} * \frac{29 - 25}{5} = 34{,}3$$