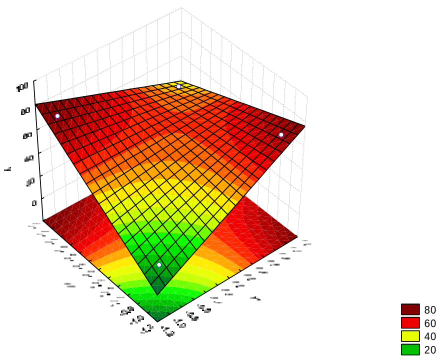
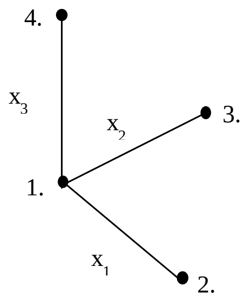
DESIGN OF EXPERIMENTS



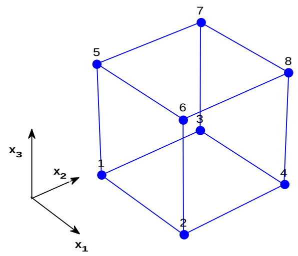
What do we want to learn?

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

2^p full factorial designs



a)



b)

one variable at a time

matrix design

Example 1

The purpose is to study the effect of apricot jam technology parameters to the quality of the product

z_1 amount of sugar 0.2 and 0.3 kg/kg

z_2 boiling time 25 and 30 min

Factors	z_1	z_2
center point z_j^0	0.25	27.5
variation interval Δz_j	0.05	2.5
upper level z_j^{max} (+)	0.3	30
lower level z_j^{min} (–)	0.2	25

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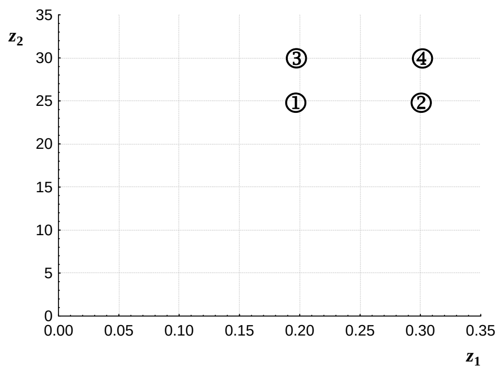
The design:

i	z_1	z_2	y
1	0.2	25	16
2	0.3	25	68
3	0.2	30	72
4	0.3	30	44



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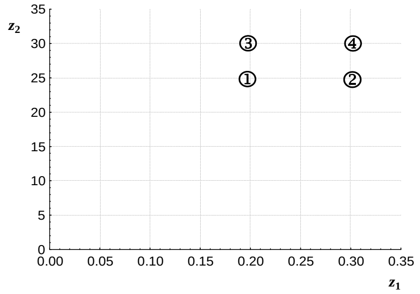
i	In natural units		Transformed (coded) units			y
	z_1	z_2	x_0	x_1	x_2	
1	0.2	25	+	-	-	16
2	0.3	25	+	+	-	68
3	0.2	30	+	-	+	72
4	0.3	30	+	+	+	44

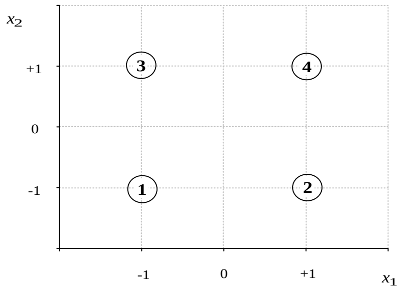
Transformation (coding):

$$x_j = \frac{z_j - z_j^0}{\Delta z_j}$$

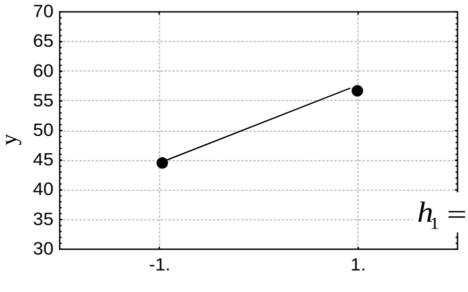
$$\sum_i x_{ji} x_{ki} = 0, \text{ if } \quad j \neq k$$

orthogonality



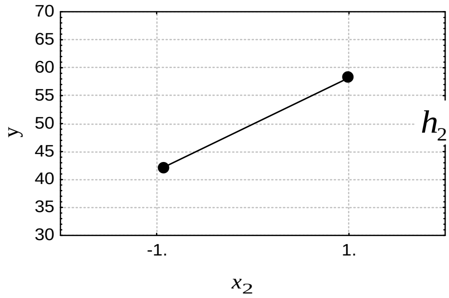


$$h_j = \left(\bar{y}_{j+}\right) - \left(\bar{y}_{j-}\right)$$



i	x_0	x_1	x_2	y
1	+	-	-	16
2	+	+	-	68
3	+	-	+	72
4	+	+	+	44

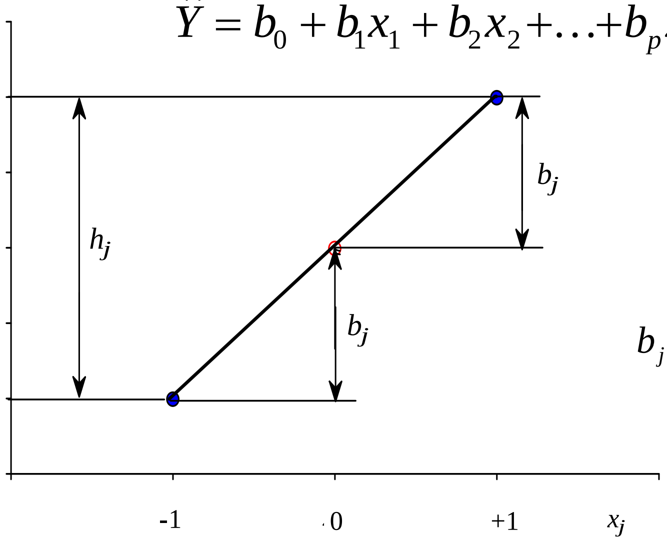
$$h_1 = \frac{68+44}{2} - \frac{16+72}{2} = 56 - 44 = 12$$



$$h_2 = \frac{72+44}{2} - \frac{16+68}{2} = 58 - 42 = 16$$

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

$$\hat{Y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_p x_p$$



$$b_j = \frac{h_j}{2}$$

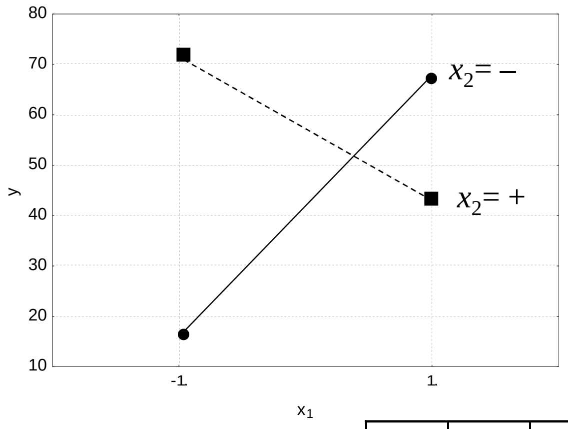
$$h_1 = \frac{68+44}{2} - \frac{16+72}{2} = \frac{-16+68-72+44}{2} = 12 \qquad b_1 = \frac{12}{2} = 6$$

$$h_2 = \frac{72+44}{2} - \frac{16+68}{2} = \frac{-16-68+72+44}{2} = 16 \qquad b_2 = \frac{16}{2} = 8$$

$$b_j = \frac{\sum_i y_i x_{ji}}{\sum_i x_{ji}^2} = \frac{\sum_i y_i x_{ji}}{N}$$

i	x_0	x_1	x_2	y
1	+	-	-	16
2	+	+	-	68
3	+	-	+	72
4	+	+	+	44

$$b_0 = \frac{16+68+72+44}{4} = 50$$



i	x_0	x_1	x_2	y
1	+	–	–	16
2	+	+	–	68
3	+	–	+	72
4	+	+	+	44

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Interaction

- If there is no interaction between x_1 and x_2 the effect of x_1 is the same at the lower and upper level of x_2 (the lines in the figure on slide 9. are parallel).
- If there **is** interaction between x_1 and x_2, the effect of x_1 is different at the lower and upper level of x_2 (the lines in the figure on slide 9. are not parallel).
- Interaction means that the **effect** of one factor depends on the **level** of the other factor(s).

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Model with interaction

$$\hat{Y} = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2$$

$$b_j = \frac{h_j}{2}$$

i	x_0	x_1	x_2	x_1x_2	y
1	+	–	–	+	16
2	+	+	–	–	68
3	+	–	+	–	72
4	+	+	+	+	44

$$b_j = \frac{\sum_i y_i x_{ji}}{N}$$

$$h_{12} = \bar{y}_{12+} - \bar{y}_{12-} = \frac{16+44}{2} - \frac{68+72}{2} = 30 - 70 = -40$$

$$b_{12} = \frac{-40}{2} = -20$$

$$b_{12} = \frac{16-68-72+44}{4} = \frac{60-140}{4} = \frac{-80}{4} = -20$$

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The estimated transfer function:

$$\hat{Y} = 50 + 6x_1 + 8x_2 - 20x_1x_2$$

Factors	z_1	z_2
center point z_j^0	0.25	27.5
variation interval Δz_j	0.05	2.5
upper level z_j^{max} (+)	0.3	30
lower level z_j^{min} (–)	0.2	25

What is the expected quality at 0.22kg/kg sugar and 27min boiling?

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The estimated transfer function:

$$\hat{Y} = 50 + 6x_1 + 8x_2 - 20x_1x_2$$

$$\hat{Y} = 50 + 6\left(\frac{C - 0.25}{0.05}\right) + 8\left(\frac{t - 27.5}{2.5}\right) - 20\left(\frac{C - 0.25}{0.05}\right)\left(\frac{t - 27.5}{2.5}\right) =$$

$$= 50 - \frac{2.5 \cdot 6 \cdot 0.25 + 0.05 \cdot 8 \cdot 27.5 + 20 \cdot 0.25 \cdot 27.5}{0.125} + \frac{2.5 \cdot 6 + 20 \cdot 27.5}{0.125} C +$$

$$+ \frac{0.05 \cdot 8 + 20 \cdot 0.25}{0.125} t - \frac{20C \cdot t}{0.125} = -1168 + 4520\, C + 43.2\, t - 160\, C \cdot t$$

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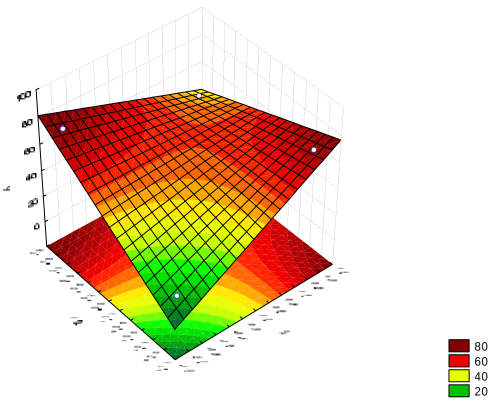
$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$$

$$\hat{Y} = 50 + 6x_1 + 8x_2 - 20x_1x_2$$



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